Probability and Statistics

Are you an average teenager? How could you find out how your answers to these questions compare with those of the average Canadian teenager?

How many hours a week do you spend playing sports?

How are you most likely to communicate with your friends?

What You'll Learn

- Understand the role of probability in society.
- Identify and address problems related to data collection.
- Use either a population or a sample to answer a question.
- Develop and implement a plan to collect, display, and analyze data.

Why It's Important

People make decisions based on data. People can make informed decisions if they understand what the data really mean and where they come from, and are confident the data are accurate.

How do your likes, dislikes, opinions, and lifestyle compare with those of your friends?

Key Words

- population
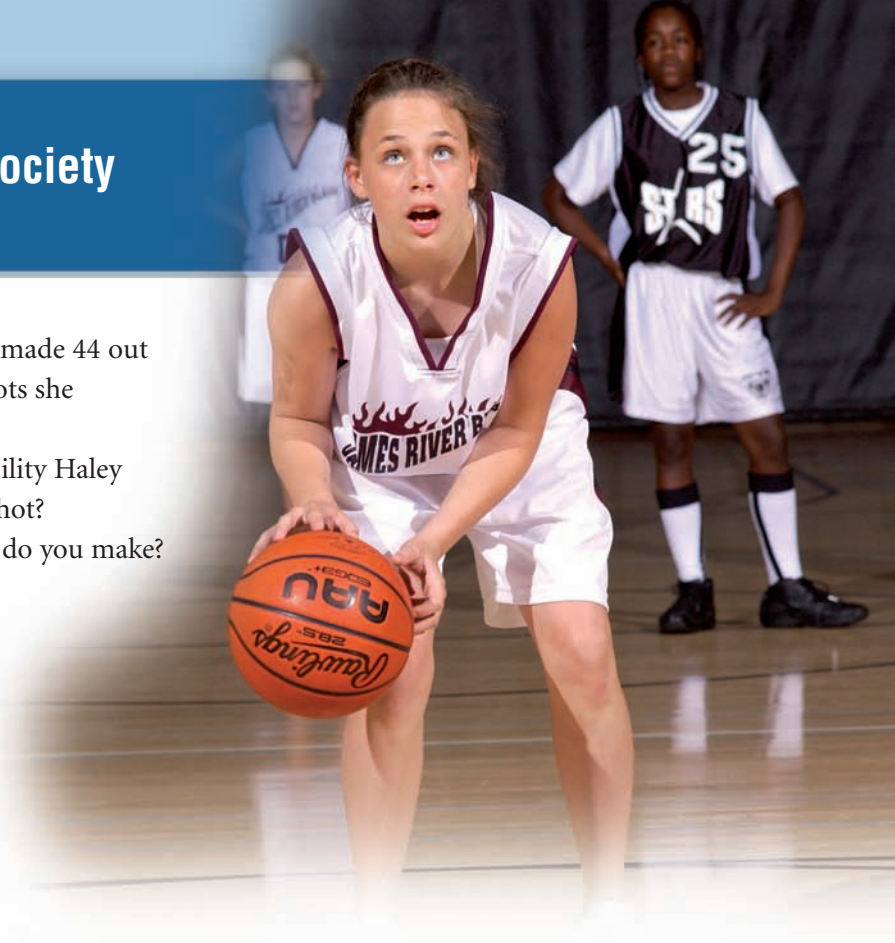- census
- sample
- valid conclusions
- rubric

FOCUS

• Explain how probability is used outside the classroom.

This season, Haley made 44 out of 50 basketball shots she attempted.
What is the probability Haley will sink the next shot?
What assumptions do you make?

Investigate

2

From the statements below, identify the different probabilities that Jean-Guy considers in a day. List an assumption associated with each statement. Explain how the situation would change if that assumption were not true.

➤ Jean-Guy noticed that, in the last month, 70% of the time the bus was 3 minutes late. So, he takes his time with breakfast today.

➤ Lately, Jean-Guy's math teacher checks homework 4 days a week. So, Jean-Guy makes sure he has time to complete his homework today.

➤ At school, Jean-Guy and his friends agree that their lacrosse team has a 95% chance of making the finals.

➤ In health class, Jean-Guy's teacher reads a magazine that claims 172 out of 1000 male smokers develop lung cancer, but only 13 out of 1000 males who do not smoke develop lung cancer.

Reflect & Share

Share your assumptions with those of another pair of students. Discuss which assumptions you think are the most likely and least likely to be true.
Which probability could have been most influenced by personal opinion?
How do you think the other probabilities were determined? Explain.

Probability refers to the likelihood that an event will occur.

By collecting and analyzing data, predictions can be made about the likelihood that a certain event will occur. For example, meteorologists study past weather data to make predictions about future conditions. A 40% probability of snow means that under similar conditions in the past it snowed 40% of the time, or 4 times out of 10.

> A probability of 40% can be expressed as 4 out of 10, $\frac{4}{10}$, or 0.4.

When you flip a coin, there is a 0.5 probability the coin will land heads up. Despite this probability, you may feel strongly that the coin will land tails up. In this case, you have made a *subjective judgment*.

Example 1 **Identifying Decisions Based on Probabilities and Judgments**

Explain how each decision is based on theoretical probability, experimental probability, subjective judgment, or any combination of these.

a) It is Ausma's experience that 4 out of 5 times the prize in the cereal box is found at the bottom of the box. So, Ausma opens the bottom of the cereal box to find her prize.

b) Two friends are rolling a die. Out of eight rolls made, a "4" came up 7 times. Amith predicts the next roll will likely not be a "4," since each number has an equal chance of being rolled. Maria decides the die is unfair since 7 out of 8 rolls revealed a "4."

▶ *A Solution*

a) Ausma's decision to open the bottom of the cereal box to find her prize is based on past experience. This is an example of experimental probability.

b) Amith's decision that the next roll will likely not be a "4" is based on theoretical probability. Amith knows that the probability of rolling each number is 1 out of 6, so the probability of rolling any number other than a "4" is 5 out of 6.
Based on previous rolls, Maria has noticed that the experimental probability of rolling a "4" is 7 out of 8. She knows that each number should have an equal probability of being rolled. So, she makes a subjective judgment that the die must be unfair.

Often, we make predictions about an outcome based on assumptions associated with a given probability.

If these assumptions change, the prediction may not match the outcome.

Example 2 — Explaining How Assumptions Affect a Probability

In past baseball games, Alice made 2 hits for every 5 times she went up to bat.

a) In the next game, suppose Alice goes up to bat. What is the probability that she will get a hit? What assumptions are you making?

b) For each assumption, explain how the predicted outcomes might change if the assumption changes.

▶ *A Solution*

a) The experimental probability of Alice hitting the ball is 2 out of 5, or 40%. We assume that the next team Alice plays against is at the same level of ability as previous teams she has played.

b) If the opposing team is more able, then Alice will probably make fewer hits. The likelihood that Alice has a hit would be less than 2 out of 5. If the opposing team is less able, then Alice will probably have more than 2 hits. In this case, the likelihood that Alice has a hit would be greater than 2 out of 5.

Sometimes a probability can be used to support opposing views.

Example 3 — Using a Probability to Support Opposing Views

Jon wants to learn how to snowboard but does not want to take lessons. His mother insists that Jon take lessons. Jon and his mother find an article that claims:

68% of snowboarding injuries occur during beginner lessons

Explain how both Jon and his mother can use this statistic to support their opinions.

▶ A Solution

68% of snowboarding injuries occur during beginner lessons. Jon's mother can argue that this statistic proves lessons are important because beginners are prone to accidents. Jon can argue that this statistic is a good reason not to take lessons because the likelihood of getting injured during the lesson is high.

Discuss the ideas

1. When you toss a coin, what assumptions are you making when you say the probability of it landing heads up is $\frac{1}{2}$?

2. Car insurance for teenagers is more expensive than for adults because the probability of an accident is greater for teenage drivers. What assumptions is an insurance company making when it charges a teenage driver more for insurance?

Practice

Check

3. Indicate whether each decision is based on theoretical probability, experimental probability, or subjective judgment. Explain how you know.

   a) The last two times Andrei won a prize at a coffee shop, he ordered a medium hot chocolate. Andrei never won when he ordered a large hot chocolate, so today he orders a medium hot chocolate.

   b) Instead of buying her own lottery ticket, Martha pools her money with the people at work to buy more tickets and increase her chances of winning.

   c) Anita boards the last car of a train because, in the past, the last car always had available seats.

   d) Doug will not travel by airplane even though experts say it is safer to fly than drive.

4. What assumptions is each person making?

   a) Based on past math quizzes, Claudia says she has a 90% chance of getting a perfect score on her next math quiz.

   b) Six times out of ten, Omar gets stuck in traffic when he leaves work. So, he calculates that his chances for getting stuck in traffic today after work are 60%.

Apply

5. The weather forecast is 70% chance of rain. Winona had planned to go canoeing. Explain how the decision she makes may be based either on probability or on subjective judgment.

6. The student council has a draw for a prize during the school dance. Lei decides not to enter the draw because all of his classmates have entered and he feels unlucky today. Is Lei's decision based on probability, on subjective judgment, or both? Explain.

7. One year, the probability of *not* recovering a stolen vehicle in Montreal was 44%. How could politicians use this fact to argue that:
   a) more money should be allotted to searching for stolen vehicles
   b) more money should not be allotted, and instead should go to different causes

8. Vanessa observes her birdfeeder at the same time each day for a week. She notes that 32 of the 100 birds which visit the feeder are cardinals. She concludes that, in general, there is a 32% probability a bird visiting the feeder will be a cardinal.
   a) What assumptions did Vanessa make?
   b) If each assumption changes, how might the predicted outcome change?

9. Kathryn read this headline:

> **Poll reveals 30% support Bradford, 70% support Choo in next election**

Kathryn says that if she polled the next 10 people she passed on the street, 7 of them would be voting for Choo.
   a) What assumption is Kathryn making?
   b) Explain what the effect might be if the assumption were not true.

10. A DNA match was found between a blood sample and a suspect. A forensic scientist testifies that there is a 1 in 7000 chance the blood sample is from someone other than the suspect. Describe how two lawyers could use this statistic to support different positions.

11. **Assessment Focus** An advertisement for acne treatment boasts:

> **4 out of 5 users noticed improvements in two days!**

   a) Explain how a teenager's decision on whether to try this acne treatment could be based on probability and subjective judgment.
   b) If the teenager does decide to try the acne treatment, what assumptions might he be making? For each assumption, explain how the predicted outcome of the treatment might change if the assumption changes.

12. a) Provide 2 examples of how statistics are used in the media to sell a product.
   b) Why do advertisers use numbers in these ads? Do you think using statistical data makes the ads more effective?
   c) For each example, list some assumptions associated with the statistic.

13. Look at newspapers, magazines, or on the internet. Give 2 examples of how politicians or environmentalists use probability.

Take It Further

14. Shaquille O'Neal's free throw percentage during one season was 62%. A teacher points out that this means each time Shaquille attempted a free throw during that season, his probability of making the shot was 62%.
A student then says:
"Shaquille either makes the shot or he doesn't. So, isn't the probability 50%?"
Explain the flaw in the student's thinking.

15. Research 2 occupations that use probability. Explain the role of probability in each occupation.

16. The annual Farmer's Almanac makes predictions about long range weather patterns. Investigate to find out how these predictions are made. What assumptions is the almanac making?

17. According to Transport Canada, in 2004, there were 34 fatalities due to air travel and 2730 fatalities due to road travel.
   a) What impression does this information give? How might this information be misleading?
   b) What additional information would you need to determine whether travelling by plane or by car is safer?

Reflect

Think of 2 decisions that could be influenced by probabilities.
What assumptions would you be making about each probability?
How might the probabilities be different if the assumptions were not true?

Math Link

Your World

Probabilities are used in risk assessment. To compare the safety of certain sport utility vehicles (SUVs) and minivans, researchers subjected both to crash tests while the vehicles moved at 56 km/h. Here are the results:

Probability of:	SUV	Minivan
life threatening head injury	16%	2%
life threatening chest injury	20%	4%
life threatening leg injury	35%	1%

How might insurance companies use this information?
How might car manufacturers use this information?

GAME

Cube Master

How to Play

1. Choose a "dealer". The dealer selects any 30 cubes. (For example, the dealer could choose 5 red cubes, 12 green cubes, and 13 blue cubes.)
   No other player should know how many cubes of each colour were selected.
   The dealer places the 30 cubes in a container.
2. Each player records a guess of how many cubes of each colour are in the container.
   Players should not share these guesses.
3. The players take turns selecting one cube from the container, then returning the cube.
   (The dealer makes sure the players cannot see what is in the container.)
   Players note which colour was selected each time.
   Stop after 10 cubes have been selected and returned.
4. Players now adjust their initial guesses by considering the colours of the cubes selected.
5. Repeat Steps 3 and 4 two more times.
6. Players compare their final estimates with the actual numbers of cubes to calculate their points.
   The player with the fewest points wins.

For example:

	Actual Number of Cubes	Player's Final Estimate	Number of Points (Difference between actual number and estimate)
Red	5	4	1
Green	12	11	1
Blue	13	15	2
Total Number of Points:			4

7. Repeat the game until everyone has had the opportunity to be the dealer.

Share your strategies with the other players.
Whose strategy worked best? Why?

You will need

- more than 30 cubes (blue, green, and red)
- a container for the cubes

Number of Players

- 4

Goal of the Game

- Use experimental probability and subjective judgment to estimate the numbers of coloured cubes in a container.

9.2 Potential Problems with Collecting Data

FOCUS

• Describe how the collection of data may be adversely affected.

Suppose your friend asks you, "Who do you think will win the Western Hockey League (WHL) playoffs?"

What might affect your answer to this question?

Investigate

Choose one person in the group to be the leader.
The leader will read the questions below, while the others write down their answers.
The leader should ask the questions exactly as they are written, without explanation.

1. How large is this textbook? (Point to this textbook when asking the question.)
2. Do you think students are working too much because of the large amount of homework assigned?
3. What is your favourite genre of music? Choose one from: Rock, Pop, Country

Share all the answers with the group.
Did you interpret the questions the same way as your group? Why?
Describe any problems with each question.
Rewrite each question to avoid these problems.

Reflect & Share

Share your rewritten questions with another group.
Discuss how the rewritten questions avoid potential problems.
How might a poorly worded question affect data collection?
What sort of wording should someone collecting data use?
What sort of wording should that person avoid?

There are several factors that might lead to problems with data collection.

Potential Problem	What It Means	Example
Bias	The question influences responses in favour of, or against the topic of the data collection.	Suppose a person asks: *Don't you think the price of a movie ticket is too high?* This person has a bias against the current ticket price, and the bias influences how the survey question is written.
Use of Language	The use of language in a question could lead people to give a particular answer.	If you ask: *Don't you think the price of a movie ticket is too high?*, the question may lead people to answer yes. A better question would be: *Do you think the price of a movie ticket is too high, too low, or fair?*
Timing	*When* the data are collected could lead to particular results.	A survey is conducted to find opinions on the need for a vehicle to have winter tires. The results may be different if the survey is conducted in August instead of February.
Privacy	If the topic of the data collection is personal, a person may not want to participate or may give an untrue answer on purpose. Anonymous surveys may help.	People may not want to participate in a study on weight if it means stepping on a scale in front of other people.
Cultural Sensitivity	Cultural sensitivity means that you are aware of other cultures. You must avoid being offensive and asking questions that do not apply to that culture.	Suppose you wanted to know the favourite method of cooking ham, and you asked: *Please circle your favourite method:* BBQ Bake Fry This question does not apply to everyone because many people do not eat ham. A better question would be: *If you eat ham, name your favourite method of cooking it.*
Ethics	Ethics dictate that collected data must not be used for purposes other than those told to the participants. Otherwise, your actions are considered unethical.	Suppose you tell your classmates that you want to know their favourite snacks to help you plan your birthday party. If you then use the information to try to sell your classmates their favourite snacks between classes, your actions would be unethical.
Cost	The cost of collecting data must be taken into account.	If you need to pay for printing the questionnaires, or to pay people to collect the data, the cost may be more than you can afford.
Time	The time needed for collecting the data must be considered.	A survey that takes an hour to complete may be too long for most people. This would limit the number of people willing to participate.

Example 1   **Identifying and Eliminating Potential Problems**

For each survey question, explain why a problem may occur and the effect it would have on the results. Suggest how each problem could be avoided.

a) A survey is conducted to find out if citizens think the local government should provide more money for youth activities. The question asked was: "Would you support an increase in taxes to create more skate parks?"

b) A survey is conducted to find out the level of school spirit. Students are polled about their level of school spirit after the soccer team wins the championship.

▶ *A Solution*

a) The use of language in the survey question could be problematic.
The question emphasizes what citizens would lose; that is, their taxes would increase. The question also downplays what citizens would gain by only mentioning skate parks, instead of a variety of activities.
Most people would probably respond by saying they would not support an increase in taxes to build more skate parks.
A better question would be: "Do you think the local government should supply more funds for youth recreational activities?"

b) The timing of the survey question could be problematic.
Since the school's soccer team just won the championship, the level of school spirit would be higher than usual.
The results of the survey may show a higher level of school spirit than if the survey was conducted at another time.
Asking students the same question a month later, when no school event is occurring, should produce more accurate results.

Example 2   **Analyzing Data Collection for Problems**

Kublu and Irniq plan to open a shop in Saskatoon that would sell traditional Inuit crafts.
To ensure Saskatoon is the best place for their business, they want to survey residents to find out how popular Inuit crafts are.
Kublu knows that they would get the most accurate results if each household in Saskatoon is surveyed, but Irniq points out that this is problematic.
Explain why.

▶ A Solution

The number of households in Saskatoon is great. Kublu and Irniq may have problems related to cost and time.

The cost of printing and mailing enough surveys for each household would be very high. Also, Kublu and Irniq should provide an envelope and stamp for each household to return the survey. This would be an additional cost.

The time it would take to print, mail, and collect the surveys for all the households would be too long.

Example 3 — Overcoming Potential Problems of Data Collection

Antonia wants to find out if there is a relationship between household income and how much people spent on Christmas presents.

Identify potential problems Antonia may encounter, and explain how she could deal with the problems.

▶ A Solution

Christmas is not celebrated by all cultures, and so the survey question does not apply to everyone. An appropriate opening question for the survey might be: "Do you celebrate Christmas?" If a person responds "No", then he or she will not need to answer the other question in your survey.

Information about income and spending habits is personal, so people may be uncomfortable revealing it. An anonymous survey would be appropriate.

The use of language may influence responses.
Examples of inappropriate or intrusive questions would be:
"How much do you make?" and "How much do you spend?"
A better question might be:
"Is the amount you spend on Christmas presents:
– greater than your weekly income?
– less than your weekly income?
– equal to your weekly income?"

Discuss the ideas

1. How could the use of language affect the data collected? Give an example.

2. Two factors that may influence data collection are *time* and *timing*. What is the difference between these?

Check

3. Name a problem with each data collection.

a) After the first week of school, your principal asks you and your friends how you are enjoying school.

b) An online magazine asks readers either to agree or disagree with the statement: "If you find a $20-bill, you turn it in."

c) Brenda asks her classmates if they think girls should not be allowed to cover their heads in school.

d) To discover the most popular kind of movie at his school, Carlos plans to ask each student what her or his favourite kind of movie is.

Apply

4. For each scenario in question 3:

a) Describe the effect each problem would have on the data collection.

b) How could each problem be overcome?

5. Parinder wanted to find out how often the computers in her school were being used. She asked students the question:
"How much time do you spend on the computer each week?"

a) How do you think her schoolmates will interpret this question?

b) How could the question be rewritten so it would more accurately reflect what Parinder wants to know?

c) Who might be interested in her findings? Why?

6. Andrew went to each class in his school and asked for a show of hands to find out how many students had ever been bullied at school. Only 2 students raised their hands.

Andrew concluded that bullying was not a major problem at his school.

a) Is this a reasonable conclusion? Explain.

b) Describe a better method for conducting Andrew's survey.

7. Trinity wants to find out how football fans feel about building a new indoor football stadium for a Canadian Football League team. She goes to the stadium to survey fans after a winning game on a warm August evening.

a) Describe how the timing of her question may influence the responses.

b) In what setting might the responses be different than those Trinity received?

8. a) Describe how each question reveals a bias of the questioner.

i) Do you think it is a good idea to use DNA tests to convict a violent criminal?

ii) Do you think gas guzzling SUVs should be banned?

iii) Do you think students should be allowed to use spell check because it automatically improves spelling?

b) Rewrite each question to eliminate the bias. Explain how your question is an improvement.

9. Rebecca was looking for a cell phone service provider. She surveyed her friends and asked who their service providers were. Based on these data, she chose the provider that her friends used more than any other.
   a) Do you think Rebecca's question reflected what she wanted to know? Explain.
   b) What questions might have helped Rebecca to make a more informed decision?

10. A fashion website is conducting a survey. Sasha answered questions about his favourite brands of clothing, then provided an email address as a login to the site in the future. Shortly after this, his inbox was full of emails advertising a new brand of clothing.
   a) Which important factor did the survey designers overlook? How is this problematic?
   b) How can the survey designers avoid this problem?

11. Provide an example of data collection where the cost and time needed to complete the collection may lead to problems.

12. a) Write 3 questions people would prefer to answer anonymously.
   b) For each question, describe what the results might be if the participants were not anonymous.

13. **Assessment Focus** Bridget wants to find out how much the average grade 9 student spends on clothes each month.
   a) Identify potential problems she may encounter related to 3 of these factors: use of language, ethics, cost, time, timing, privacy, cultural sensitivity
   b) For each potential problem in part a, explain how Bridget could avoid the problem.

14. a) Describe 2 possible data collections that might be problematic because of the time of year they are conducted.
   b) Suggest a better time that each should be conducted.

Take It Further

15. Common methods of surveying are by personal interviews, over the phone, or by email. Identify potential problems associated with each method of surveying.

16. a) Why might questions about Hanukkah be culturally sensitive?
   b) Think of 3 more topics that might be culturally sensitive. Explain why.
   c) Design a culturally sensitive survey question about one of the topics in part a or b. Explain how you would collect the data to address the cultural sensitivity.

Reflect

Why is it important to identify and overcome sources of potential problems in data collection?

FOCUS

• Select and defend the choice of using a population or a sample.

To estimate the number of salmon in a river, biologists use a strategy called *mark and recapture*. At one place in the river, biologists capture some fish. Each fish is marked with a tag, then released into the river. At a different place in the river, biologists recapture fish. They track the numbers of marked and unmarked fish caught. They can then estimate the salmon population.

Investigate

2

Each pair of students will need 50 small pieces of paper.

➤ Create a population of 50 fish by labelling each piece of paper with either "F" for female or "M" for male.

➤ Record the percents of female and male fish in your population.

➤ Fold the pieces of paper and place them in a box.

Trade populations with another pair of students.

➤ Choose a sample of 10 fish from the other pair's population. Record the numbers of male and female fish in your sample. Use this to estimate the percents of male and female fish in the population.

➤ Repeat the preceding step by choosing samples of 20 fish, and then 40 fish.

➤ Did your estimates of the percents of male and female fish change as the sample size changed? Explain.

Reflect & Share

Compare your estimates with the actual percents recorded by the other pair of students. How did your estimates compare with the actual percents? In general, how did the estimates change as the size of the sample increased?

Connect

When collecting data, the **population** is the group about which you are getting information. A **census** is conducted when data are collected from each member of the population. For example, suppose you test game consoles made in a factory for defects, then *all* the game consoles made in the factory are the population. If you test each game console, then you have conducted a census.

A census can be costly, time consuming, and difficult or impossible to complete. So, a census is only used when an issue is important or when the population is small.

If a census is not feasible or necessary then data are collected from a small portion, or **sample**, of the population. When the sample chosen is representative of the population, the data collection provides **valid conclusions**. For example, testing 100 game consoles out of 1000 made each day is a sample. If those consoles tested represent the typical quality of consoles made in the factory, the conclusions of the data collection will be valid.

Care must be taken when determining the appropriate size of the sample. If the sample is large, the data collection could be costly or time consuming. If the sample is small, then it may not be representative of the population.

Example 1   Explaining Why Data Are Collected from Populations

In each case, explain why a population was surveyed instead of a sample.
a) To determine the average number of siblings of his classmates, Carlos surveyed each person in the class.
b) Every 5 years, Statistics Canada conducts a census. One question in the survey is used to determine the ages of the people in each household.

▶ *A Solution*

a) Carlos knows that surveying the entire population will produce exact results, rather than estimates. So, he chose to survey the entire population, the whole class, because it would not take long or cost him anything.
b) A census was completed because of the importance of the question. The government requires data about the ages of Canadians so that it can budget for services such as day-care centres, schools, and senior citizens' homes.

Example 2 | **Reasoning Why and When Samples Should Be Used**

The student leadership team is planning a school dance. To attract grade 9 students to the dance, the team decided to collect data about the preferred music of the grade 9 students. The team set up in the hallway to collect the data. By the end of the day it had surveyed 73% of the grade 9 students.

a) Why do you think the data were collected from a sample instead of the entire population?

b) Will the opinions of the sample likely reflect those of the population? Explain.

▶ *A Solution*

a) There was probably not enough time or people available to ask all grade 9 students. It would also require a lot of effort to find each grade 9 student, especially with absences.

b) Since the majority of students, 73%, were asked, it is likely that their opinions will reflect those of the entire population.

Example 3 | **Identifying and Critiquing the Use of Samples**

In each case, identify if data were collected from a sample or a population. Wherever a sample was used, explain if you think the conclusion would be valid.

a) A province considers banning cell phones in all of its schools. To determine the opinions of students on this issue, you poll each student in your school.

b) To determine which politician is expected to win the municipal election, every person over 18 and who is eligible to vote in the election is polled.

c) To determine the average lifetime of a type of light bulb, 150 light bulbs were selected randomly from the production line and tested.

▶ *A Solution*

a) Sample: The population is all students of all schools in the province. By asking only the students in your school, your results are based on a sample. If the students in your school do not represent typical students in the province, the conclusion will not be valid. For example, if all students in your school own cell phones, your conclusion would probably be not to ban cell phones. However, not every student in the province owns a cell phone. So, your results would not be representative of the population.

b) Population: All possible voters are polled.

c) Sample: Since not all light bulbs were tested, the results are based on a sample. It would not make sense for the whole population to be tested, since all light bulbs would be destroyed in the process. There would be no light bulbs left to sell. Since a fairly large number of light bulbs were tested, the results will likely give a good estimate of the lifetime of a light bulb. So, the conclusion about the lifetime of a light bulb is likely valid.

Discuss the ideas

1. What factors do you need to consider when you collect data from either a population or a sample?

2. What does "valid conclusion" mean? Provide an example where the conclusion based on a sample is not valid.

Practice

Check

3. In each case, describe the population.
   a) The management team of a shopping mall in Comox wants to know how to attract more people between the ages of 13 and 25 to the mall.
   b) A juice company wants to determine the average volume of juice in a 1-L carton.
   c) A board of education wants to find out which schools need renovations.
   d) The government wants to determine the average age of First Nations people in Nunavut.

4. In each case, are the data collected from a census or a sample?
   a) To determine the favourite TV show of grade 9 students in a school, all grade 9 students in the school are surveyed.
   b) To find out if customers of a chain of coffee shops are happy with the service, some customers in every shop were surveyed.

Apply

5. Identify the population you would sample to find out opinions on:
   a) bus fares   b) the GST   c) cost of day care
   d) emergency room wait times

6. Courtney surveys her friends and finds that 68% of them have an MP3 player. She reports that 68% of the grade 9 students have an MP3 player. James surveys the entire grade 9 population and discovers that 51% have an MP3 player.
   a) Whose conclusion is more likely to be valid? Explain.
   b) Why might the other student's conclusion not be valid?

7. For each situation, explain why data are collected from a sample and not a census.
  a) to determine the number of hours an AAA battery will last in a calculator
  b) to determine the number of First Nations children in Canada who speak Cree

8. Should a census or sample be used to collect data about each topic? Explain your choice.
  a) the effectiveness of a new suntan lotion
  b) the popularity of a fruit-flavoured yogurt
  c) the number of grade 9 students in your school with braces
  d) the number of your friends who like to play computer games

9. In each case, do you think the conclusion is valid? Justify your answers.
  a) Irina surveyed 20 students to find out if they eat breakfast. All the students said yes. Irina concluded that everyone in the school eats breakfast.
  b) To test for pesticide pollution, a scientist collects and tests one vial of water from a river. From the results, a local newspaper reporter concludes that there are dangerous levels of pesticide in the river.

10. **Assessment Focus** Suppose you are the manager of a high school cafeteria. You want to create a new breakfast and lunch menu for the students.
  a) What population are you interested in surveying?
  b) Would you survey a sample or population? Explain.
  c) If you had to use a sample, what would you do to make sure your conclusions are valid?

11. In each case, provide an example and justify your choice.
  a) Collecting data from a population, rather than a sample, is more appropriate.
  b) Collecting data from a sample, rather than the entire population, is more appropriate.

Take It Further

12. a) Describe a situation where a sample:
   i) represents a population
   ii) does not represent a population
  b) What changes would you make to the sample in part ii so that the conclusions would be valid?

13. Every 5 years, the *Census of Agriculture* is sent to every farm household across Canada. This census collects data on topics such as crop area, livestock, farm labour, machinery, and expenses. Choose one of these topics. Explain why you think it is important enough for the government to conduct a census.

Reflect

Describe when to use a census and when to use a sample.

Statistics Canada is a government agency that collects data on Canadian citizens. *Census at School* is an international online project that engages students from grades 4 to 12 in statistical enquiry. Data on students from 8 to 18 years old can be found at the *Census at School* website.

You can use *Census at School* to find data about Canadian youth under headings such as:

What is your favourite subject?
How much pressure do you feel because of schoolwork?
What is your favourite physical activity?

There is a link to data from other participating countries such as South Africa and New Zealand.

To use *Census at School*, follow these steps:

1. Open the website. Your teacher will give you the address. Select the appropriate language.

2. You will see the *Census at School* homepage. Click on *Data and results* located in the table on the left of the screen.

3. Under *Canadian summary results*, you can access data collected over the past several years. Select the latest summary results.

Canadian summary results

Student responses from across the country are collected throughout the school year and analysed during the summer. Then summary data tables are published in the fall.

- Summary results 2007/2008 (including provinces and territories)
- Summary results 2006/2007 (including provinces and territories)
- Summary results for 2005/2006 (including provinces and territories)
  - Highlights: What kids said in the last survey
- Summary results for 2004/2005 (including provinces and territories)
- Summary results for 2003/2004

Note: These results can be viewed by provinces and territories by selecting the *Provinces and territories* link at the top of the page.

Canadian summary results for 2007/2008

- Canada
- Provinces and territories

4. Use the site to answer these questions.

  a) What is the most popular mode of transportation to school for Canadian youth? Is this the most popular mode of transportation for each province or territory? Explain.

  b) What are the two most popular methods of communication for Canadian students?

  c) Which type of charity would most students support if they had $1000 to donate?

  d) What percent of students have been bullied 10 times or more in the last month in Manitoba?

5. To find data from other countries, click on the *International project* link on the left of the screen. Next, click on the *CensusAtSchool* link. Then on the next screen click on the link provided.

6. Select the United Kingdom (UK). Click on the link to *Results and Data*. Then, select the *Phase 7 Results* link. What do 14-year-olds feel is the most important issue facing the UK today?

Results and Data
- See the data presented in a variety of formats, eg tables, spreadsheets, graphs.
- Get a RANDOM SAMPLE of the raw census data.
- Access all the different Questionnaires including language versions.

Check

Return to the data for Canadian youth.

1. Select a topic that interests you, and report on your findings.

2. How many elementary and secondary students have participated in Canada's *Census at School*? Do you think this sample would produce valid conclusions? Explain.

9.1

1. Before a security company hires someone, that person must pass a lie-detector test. Suppose that a lie detector has a 0.9 probability of identifying a lie.
A person being tested thinks that if he lies 10 times, 9 of those lies will be detected.
a) Name one assumption the person is making.
b) Explain how the predicted outcome might change if the assumption changes.

2. Due to global warming, the West Antarctic Ice Sheet (WAIS) could melt and raise sea levels. Some scientists think there is a 1 in 20 chance that WAIS will collapse in the next 200 years. Explain how this statistic could be used to support opposing positions about the effects of global warming.

9.2

3. Ca Bol surveys a group of people to find out how they feel about students listening to music while studying.
a) Write a question Ca Bol could use to influence:
   i) the responses in favour of students listening to music while studying
   ii) the responses to oppose students listening to music while studying
b) Write a question that does not show a bias. Explain how this question is more suitable than the questions in part a.

4. Suppose your teacher conducts a survey in class about student smoking.
a) What problems might arise?
b) How would these problems affect the data collected?

5. Ahmed wanted to find out if a person's years of post-secondary education is related to how much the person earns.
a) Describe problems Ahmed might have to overcome related to:
   i) privacy
   ii) cultural sensitivity
   iii) use of language
   iv) cost and time
b) Describe the effect each problem may have on Ahmed's results.

6. Describe a situation where the timing of a question may influence the responses.

9.3

7. Which students in your school would you survey for their opinions on each topic?
a) the quality of cafeteria service
b) the cost of a gym uniform
c) the number of student parking spaces
d) the school spirit at football games

8. For each situation, explain why data were not collected by a census.
a) the number of Canadian families with internet access
b) the average cost of DVD players
c) the average mass of a Northern pike in Misaw Lake, Saskatchewan

9. For each topic, would you collect data using a census or a sample?
Justify your choice.
a) to determine the average height of a grade 9 student in your class
b) to determine the reaction to new traffic laws in your province or territory

9.4 Selecting a Sample

FOCUS

• Understand and choose an appropriate sample.

When we cannot survey an entire population, we choose a sample from the population.

When a political party wants to determine if its candidate is likely to win the next territorial election, it may conduct a telephone survey of a sample of voters. How could the party ensure that the sample is representative of the population?

Investigate

2

Suppose a school considers making the cafeteria food more healthy. The school would like you to determine the reactions of the school population.

List 3 ways you could select the sample of people to be questioned.
Discuss and record the advantages and disadvantages of each way.
Choose the most appropriate way of selecting your sample.

Reflect & Share

Share your 3 ways of selecting a sample with another pair of students.
Did you come up with any of the same ways?
Which ways had similar advantages and disadvantages?
Discuss and select the best way from all the possibilities.
Justify your choice.

Here are some common sampling methods:

Simple random sampling
Each member of the population has an equal chance of being selected.
For example, to select a random sample of 5 students from your math class, each student is assigned a number and 5 numbers are drawn from a hat.

Systematic or interval sampling
Every nth member of the population is selected.
This method is often used in manufacturing; for example, every 20th product in an assembly line is tested for quality. If the item is destroyed or unusable after being sampled, then the sample is a *destructive sample.*

Cluster sampling
Every member of each randomly chosen group of the population is selected.
For example, each grade represents a group of the school population. One grade in your school is chosen randomly, and all students in that grade are selected.

Self-selected sampling
Only members who are interested and volunteer will participate.
For example, if a radio station conducts a telephone survey, only people who are interested will call.

Convenience sampling
Only members of the population who are convenient to include are selected.
For example, for a survey about grocery shopping habits, people in a grocery store are approached and questioned.

Stratified random sampling
Some members from each group of the population are randomly selected.
For example, 5 randomly chosen students from each grade in a school could be selected, even if each grade has a different number of students.

Example 1   **Identifying Appropriate Samples**

The student leadership team wants to find out if students would like the cafeteria to have longer hours. Several sampling methods were suggested. Explain whether each sample is appropriate.

a) Every student's name is put into a box, and 100 names are selected randomly to be surveyed.

b) Every 5th person entering the school is selected.

c) Each person on the leadership team asks her or his friends.

d) An announcement is made asking anyone who wishes to participate to fill in a ballot.

▶ *A Solution*

Sampling Method	Is the Sample Appropriate?
a) Simple random sampling	The sample is appropriate because every student has an equal chance of being selected.
b) Systematic sampling	The sample may or may not be appropriate depending on when you ask students. If you ask students who arrive early in the morning, then these students may appreciate the cafeteria having longer hours. The opinions of these students would likely not be representative of the entire student population.
c) Convenience sampling	The sample is likely not appropriate because friends often have similar views on issues.
d) Self-selected sampling	The sample is likely not appropriate because only students who have strong opinions about this topic may respond.

Example 2   **Choosing Appropriate Samples**

A company packages boxes of granola bars. The quality-control manager inspects the first 5 boxes each morning to ensure that each has the same number and types of granola bars.

a) Is this a good way of ensuring quality control? Explain.

b) Suggest 2 other methods of sampling that would be appropriate. Explain why each is appropriate.

▶ *A Solution*

a) This may not be a good way of ensuring quality control because the people working on the assembly line may be more alert in the morning. So, the boxes filled in the mornings may pass inspection. However, the boxes made later in the day, which may not meet the manager's standards, are never inspected.

b) Systematic sampling would allow the manager to inspect several boxes throughout the day. For example, each 50th box could be inspected. Simple random sampling throughout the day would also be appropriate because it ensures each box has an equal chance of being selected.

1. Which sampling methods are least likely to produce valid conclusions? Explain.

2. Which sampling methods are most likely to produce valid conclusions? Explain.

Practice

Check

3. Identify a potential problem with each sampling method.
   a) Suppose you want to know whether most people enjoy shopping. You survey the shoppers at a local mall.
   b) The cook in the school cafeteria surveys the teachers to find out which items to sell.
   c) To determine public opinion on the effectiveness of the local police force, residents in the area with the greatest crime rate are surveyed.
   d) To find out about the exercise habits of Canadian teenagers, a fitness magazine asks its readers to email information about their exercise habits.

Apply

4. Explain whether each sample is appropriate. Justify your answer.
   a) A TV show asks viewers to text their opinions about the decreased speed limit in town.
   b) To determine if customers are pleased with the service in a restaurant, every 8th customer is polled on a given day.
   c) Fifty student ID numbers were randomly selected by a computer. The students with these ID numbers were surveyed about a new school policy.
   d) Ten students were randomly selected from each grade to estimate how many students in the school cycle to school.

   e) To determine if all physical education students would prefer to go skiing or skating on a field trip, one gym class was randomly selected from a list and each student in the class was polled.

5. a) In each case, will the selected sample represent the population? Explain.
      i) To find out if the arena should offer more public skating times, a survey is posted on a bulletin board in the arena and left for patrons to complete.
      ii) To find out the favourite breakfast food of grade 9 students, a survey of 300 randomly-selected grade 9 students was conducted.
      iii) To find out if the soccer league should buy new uniforms for the players, 20 parents of the students in the soccer league were surveyed.
   b) If the sample does not represent the population, suggest another sample that would. Describe how you would select that sample.

6. Describe an appropriate sampling method for each situation. Justify your answers.
   a) The Prime Minister wants to know citizens' opinions about the new budget.
   b) The school newspaper wants to poll students to predict who will be elected student president.

7. To determine citizens' view of new parking fines, the mayor invites listeners to call in during a radio show. Do you think the results will accurately reflect the opinions of all citizens? Explain.

8. **Assessment Focus** Suppose you want to find out how people feel about lowering the age at which teens can drive.

    a) Describe a sampling method that would not lead to valid conclusions. Justify your choice.

    b) Describe a sampling method you might use, and justify your choice.

9. For each topic, identify a sample of people whose opinions would bias the survey results. Explain your choice.

    a) whether fur from animals should be used for coats

    b) whether households should be fined for not recycling

10. A survey reports: Fifty Canadians say that the most important issue Canadians face is global warming.

    a) Do you think this sample is representative of the population? Explain.

    b) How might this sample have been selected?

c) Suppose you were to repeat the survey. How would you select a sample? Explain how your choice of sample would represent the population.

Take It Further

11. Some sampling methods produce invalid conclusions more often than they produce valid conclusions. Which sampling methods do you think fit this description? Why do you think these sampling methods are still used?

12. a) Explain how you might obtain each sample.

    i) a simple random sample from the school population

    ii) a systematic sample of cell phones from a factory

    iii) a cluster sample of teenagers from your town

    iv) a stratified random sample of apple trees in an orchard

    b) Suggest a topic of data collection for each sample in part a.

Reflect

When you select a sample to represent a population, what factors must you consider?

Using Spreadsheets and Graphs to Display Data

You can use a graph to display your data in a way that is clear and easy to understand.

Spreadsheet software can be used to record and graph data.

These data come from the *Census at School* website:

Which method do you use most often to communicate with friends?

Method of communication	Girls	Boys	All students
			%
Internet chat or MSN	36.11	35.26	35.7
In person	30.02	35.51	32.65
Telephone (land line)	15.61	13.5	14.6
Cell phone	8.91	7.66	8.31
Text messaging	6.59	3.77	5.23
E-mail	1.73	2.20	1.96
Other	1.03	2.11	1.55

Notes: Secondary students only.
Methods of communication appear in order of frequency for all students.
Source: Statistics Canada, Census at School, 2006/2007.

1. Enter the *Method of communication* and the percent of *All students* into columns and rows.

2. Highlight the data including the column heads. Click the graph/chart icon. Select the circle graph, which is sometimes called a pie chart. Label the graph and all sectors of the circle.
   Your graph might look like this:

| Method of communication	All students
Internet chat or MSN	35.7
In person	32.65
Telephone (land line)	14.6
Cell phone	8.31
Text messaging	5.23
E-mail	1.96
Other	1.55

Method of communication - All students
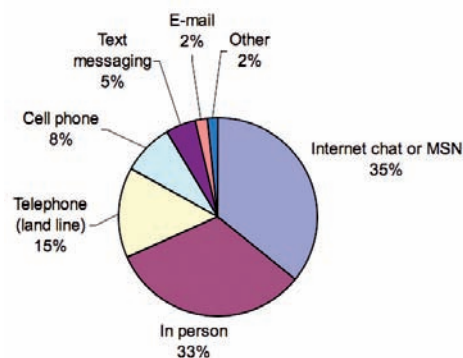
3. You can also display the data using a bar graph. Create a *Vertical bar graph,* sometimes called a *Column graph* for the data. Label the graph and axes. Experiment with the scale of each axis to most clearly display the data.

4. To display the data for *Boys*, *Girls*, and *All students*, you can make a multiple bar graph.

Enter the data for *Boys* and *Girls* into the next two columns of the spreadsheet. Now highlight all the data and create a bar graph.

Your finished graph might look like this:

Method of communication	All students	Boys	Girls
Internet chat or MSN	35.7	35.26	36.11
In person	32.65	35.51	30.02
Telephone (land line)	14.6	13.5	15.61
Cell phone	8.31	7.66	8.91
Text messaging	5.23	3.77	6.59
E-mail	1.96	2.2	1.73
Other	1.55	2.11	1.03

Method of communication
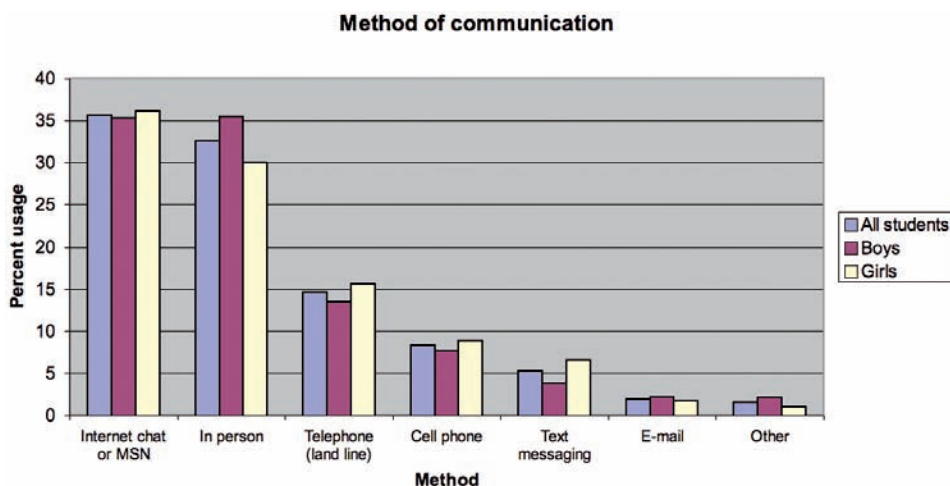
Check

1. List one advantage and one disadvantage of displaying data using each type of graph above.

2. These data are from *Census at School*:

How long does it usually take you to travel to school?

Minutes	Elementary	Secondary
	%	
Less than 10	38.36	18.96
10 to 19	31.83	31.92
20 to 29	12.55	17.96
30 to 44	10.65	16.83
45 to 59	4.16	7.71
60 or more	2.44	6.61

Source: Statistics Canada, Census at School, 2006/2007.

Use an appropriate graph to display the results for all students. Justify your choice of graph.

How Can I Assess My Work?

I can design a **rubric**. A rubric helps me to see how well I understood the task and how good I am at communicating what I know. It lists the content and quality needed for a task; these are called the *criteria* of the task.

My first step to create a rubric is to determine the criteria of the task. Suppose I have to write an article for my school newspaper.

The criteria I would look for in the article are:

- Accurate information
- Well organized
- An eye-catching title
- Correct spelling and grammar

I then assess each criterion using one of 4 *levels of achievement*:

- Not yet adequate
- Adequate
- Proficient
- Excellent

I create a grid:

	Not yet adequate	Adequate	Proficient	Excellent
Accurate information				
Correct spelling and grammar				
Well organized				
Eye-catching title				

I then include what I think needs to be done to achieve each level.

	Not yet adequate	Adequate	Proficient	Excellent
Accurate information	uses few facts that are not enough to explain the topic (1-2 facts)	uses some facts to explain the topic (3-4 facts)	uses most of the available facts to explain the topic (5-7 facts)	uses all the available facts to explain the topic (8 facts)
Correct spelling and grammar		has some spelling and grammatical errors (4-6 errors)	has few spelling and grammatical errors (1-3 errors)	has no spelling or grammatical errors (0 errors)
Well organized	ideas are not in an order that makes sense	ideas are partly in an order that makes sense		all ideas are in an order that makes sense
Eye-catching title	attempts to write a title, but it is not clear and will not attract readers	writes a title that is clear, but may not attract many readers	writes a title that is clear and effective at attracting many readers	writes a title that is very clear and is outstanding at attracting all readers

Check

1. What descriptions might go in the grey areas?

Apply

2. a) Create a rubric to evaluate your day at school. Use the criteria below.

	Not yet adequate	Adequate	Proficient	Excellent
Homework was done				
On time for classes				
Paid attention				
Helped others				

b) Trade rubrics with a classmate. Suggest how to improve your classmate's rubric.

9.5

Designing a Project Plan

Have you been stopped in the shopping mall by a person with a clipboard and been asked to answer questions for a survey? Have you ever answered the telephone at home in the evening and been asked if you would take part in a survey? Or, have you completed an online survey? If so, what types of questions were you asked?

Investigate

Suppose your school board would like to know if there is a relation between the number of hours a student works at a part-time job and her or his academic success. Design a plan that the school board could follow to collect the data.

Reflect & Share

Share your plan with that of another group.
How are your plans similar? How are your plans different?
Do your plans avoid potential problems? Explain.
If not, make adjustments to your plan.

Connect

Here are 5 possible steps to consider when you design a plan for data collection:

1. **Prepare a question.** The wording should avoid biases, and be culturally sensitive. If the survey question is personal, the participants should be anonymous.

2. **Identify the population, and possibly choose a sample.** If you select a sample, ensure it represents the population. Consider the time and cost involved in collecting data from your population or sample.

3. **Collect the data.** Consider the timing of your data collection: does it avoid potential problems?

4. **Analyze and display the data.** Choose an appropriate display for the data, such as a table, circle graph, bar graph, or line graph.

5. **Design a rubric.** This should help you evaluate the important components of your project.

Suppose a frozen yogurt company considers adding a new flavour to its menu.
Decide how to conduct a survey to determine whether the new flavour should be added to the menu.

▶ *A Solution*

Follow the steps:

1. The survey question might be:

> *Rate the taste of this new flavour of frozen yogurt on a scale of 1 to 5.*
>
> | *1* | *2* | *3* | *4* | *5* |
> |---|---|---|---|---|
> | *Strongly Dislike* | *Dislike* | *No Opinion* | *Like* | *Strongly Like* |

2. The population is all people who eat frozen yogurt and may purchase it from this company. A sample of the population might be every 10th customer who comes into a store one day from when it opens to when it closes.

3. To collect the data, the company must have a server who offers every 10th customer a free sample, poses the survey question, and records the responses.

4. To analyze the data, determine the number of people who chose each rating. This can be displayed on a circle graph. Then calculate the mean rating.

5. Evaluate the process you followed to determine if the data are valid and that you have accounted for all possible biases.

Discuss the ideas

1. Why is it important to plan before you collect data?

2. How can a rubric help you reflect on your plan?

Practice

In this lesson, you will prepare a project plan for your data collection. In the Unit Problem, you will carry out the data collection, analyze and display your results, then draw conclusions.

Developing a Project Plan

3. a) Choose a topic that interests you. Prepare a question you want to answer about that topic.

b) Explain how the wording of your question avoids bias.

c) Is your topic a sensitive one for different cultures? If so, how will you deal with this?

d) Test your question on a classmate. Make any necessary changes.

4. a) Describe the population for your data collection.

b) Will you collect data from the population or a sample? If you work with a sample, how will you select the sample to ensure it represents the population?

c) Explain how your choice of population or sample takes into account the time and cost of the project.

5. Explain how you will collect the data. As part of your answer, discuss:
- how the timing of your data collection will not cause a problem
- any privacy considerations

6. How might you display your data? Justify your choice.

Creating a Rubric for Your Project

7. Create a rubric for your project. In the Unit Problem, you will collect the data and present your findings. Your rubric must assess the following criteria:
- the survey question
- the choice of sample or population
- how the data were collected
- the display of your data
- the conclusions you made
- your presentation

Your rubric should have 4 levels of achievement.

Assessing Your Plan

8. a) Trade project plans with a classmate. Look for potential problems in your classmate's plan. Suggest ways of improving the plan.

b) Incorporate your classmate's comments into your plan.

Reflecting on Your Plan

9. People often make predictions about the results of their data collection. Sometimes these predictions are based on data from the past or personal experience. Make a prediction about your results. Explain why you think it might be true.

10. Why is this topic of interest to you? Who else might be interested in the results?

Reflect

Which step in developing your plan did you find most challenging? Explain.

Math Link

Your World

Every five years, Statistics Canada completes a census by collecting data from every household in Canada. Statistics Canada also requires one in five households to provide more detailed information about themselves. These data are used to help us better understand our country, including its natural resources, educational needs, and the economic situations of people living in various regions. Since the same or similar questions are asked every 5 years, the data can show what changes have taken place in our population over time.

Study Guide

Probability

◗ Probability is the likelihood an event will occur. For example, a weather forecast says that the probability of rain is 60%. This assumes that the predicted weather conditions do not change. If they do change, then the likelihood of rain may also change.

◗ Decisions based on probabilities may be a combination of theoretical probability, experimental probability, and subjective judgment. People may make different decisions based on one probability. For example, one person may consider a 60% probability of rain as being too high, and cancel a planned outdoor event. Another person may say that a 40% probability it will not rain is good enough to proceed with the event.

Collecting data

◗ Problems may arise if a person does not consider:
  – bias
  – use of language
  – ethics
  – cost and time
  – timing
  – privacy
  – cultural sensitivity

◗ The population is the group from which you are getting information.

◗ When a census is conducted, data are collected from the entire population.

◗ When data are collected from only part of the population, a *sample* is used. This sample must be representative of the population.

◗ Valid conclusions are obtained when the sample results represent those of the population.

9.1 **1.** Two weeks before a municipal election, a survey produced these data about voters' preferences for a new mayor.

Preference for Mayor	Number of Votes
Vivian Rogers	19
Fred Yao	11
Mustafa Abaz	34
Undecided	6

   a) Based on these data, predict which candidate will win.
   b) What assumptions are you making? For each assumption, explain what would happen if the assumption changes.

2. A volleyball team has won all 5 games it played this season.
   Darrell thinks that the team will lose its next game because he believes their winning streak cannot last.
   The players on the team believe that the team has a 100% chance of winning the next game.
   Explain how probability and subjective judgment may be influencing these opinions.

3. A local hospital is raising money by selling lottery tickets. The chances of winning are advertised as 1 in 3. Explain how someone can use this probability to support:
   a) purchasing a ticket
   b) not purchasing a ticket

4. Find an example where statistics are used in an advertisement to try to convince consumers that one product is better than another. Suppose the statistic is true. What assumptions are you making about how the data were collected?

9.2 **5. a)** In each case, identify any problems.
   i) To determine people's reactions to a possible increase in minimum wage, a student asked: "Don't you agree that minimum wage should increase?"
   ii) For a class project, a student needed to determine if college students were maintaining healthy weights. After the student completed the project, she gave a weight loss company the addresses of any overweight participants.
   iii) To test the safety of its deluxe car model, a company subjects many cars to different crash tests.
   iv) To determine if residents would support the construction of a community outdoor pool, you survey residents by going door-to-door in November.
   b) Describe the effect each problem might have on the results of the data collection.

6. Adila wants to know which digital camera is the best buy.
   a) What do you think "best buy" means? Design a question that will give Adila the information she wants.
   b) Explain how your question avoids bias.

7. For a school project, Raheem wants to find out if there is an increase in teen pregnancy in Canada. Describe the effect of problems Raheem might encounter related to:
   a) privacy
   b) cultural sensitivity
   c) use of language

8. a) Provide an example of a question that could lead to problems because participants are only allowed to choose from 3 possible answers.
   b) Reword the question to avoid the potential problems.

9. Provide an example of data you might want to collect, but where the cost and time involved might be problematic.

9.3 10. As an operator of a skydiving school, would you suggest a census or a sample when inspecting parachutes for excess wear? Explain.

11. Leah wants to test the lifespan of different brands of batteries.
    a) List reasons why Leah would use a sample for her data collection.
    b) Suggest how she might choose her sample to ensure it represents the population.

12. a) Explain why a sample might be used in each survey.
      i) You wish to find out the most popular sport for teens your age.
      ii) You wish to find out the most popular Internet provider in your area.
    b) Provide an example when a census would be more appropriate for collecting data than a sample.

9.4 13. A TV show surveys viewers to determine the popularity of its singers. At the end of each show, viewers cast votes for their favourite singers by calling in at $0.75 per call. Do you think the opinions of the sample will reflect those of the population? Explain.

14. Discuss whether each sampling method should lead to valid conclusions.
    a) A car company subjects every 200th vehicle it manufactures to crash tests.
    b) Six students from each grade are selected randomly to complete a survey about which extracurricular activities the school should offer.
    c) A juice company sets up a booth in a local mall and allows anyone who wishes to participate in a taste test.

15. Describe an appropriate sampling method for each case. Justify your choice.
    a) to predict which political party will win the next election
    b) to determine which brand of tennis racket is preferred

9.5 16. Adam wants to find out what brand of chewing gum is recommended by most dentists.
    a) Write an appropriate survey question Adam could use.
    b) Choose an appropriate sample. Explain your choice.
    c) Explain how Adam could collect the data and display the results.
    d) Explain how Adam could express his results as a probability.

1. A coin is tossed 5 times and each time it lands heads up. The coin is to be tossed again. Shawnie says the coin will land heads up. Owen says the coin will land tails up. Jovana says the coin is equally likely to land heads up as tails up. How might these students have made their predictions?

2. Hannah's hockey team won its last 7 out of 8 games. Hannah calculates that the team's probability of winning the next game is $\frac{7}{8} = 0.875$.
   a) What assumptions is Hannah making?
   b) For each assumption, explain how the predicted outcome might be affected if the assumption changes.

3. Manroop wants to survey Canadians to determine how happy or depressed they are.
   a) Describe how the timing of Manroop's survey may influence her results.
   b) Explain how privacy may be a factor in this survey.
      What should Manroop do to ensure the privacy of the participants?
   c) Manroop designs the following question:

   On a scale of 1 to 6, how satisfied are you with your life? Circle a number.
          1   2   3   4   5   6
      Dissatisfied ---------------------------- Satisfied
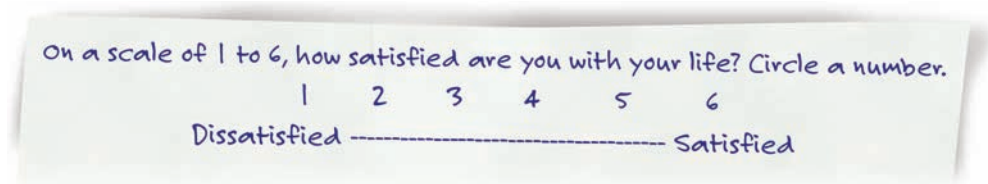
   What problems might Manroop encounter with this question?
   What effects would those problems have on her data?

4. Provide an example of a situation where:
   a) collecting data from a sample is more appropriate than a census
   b) a sample may not result in the same conclusions as a census
   Justify each example.

5. For each case, explain how you would select a sample.
   a) to test the water quality in your school
   b) to determine the most popular brand of toothpaste used by students at your school
   c) to measure the average mass of backpacks of students at your school

6. Emile starts a petition to ask the municipal government to allow all stores to open at 9 A.M. on Sunday morning. What problems might he encounter as he solicits signatures? Describe the effect each problem would have on his results.

What Can You Discover about the World around You?

You will collect data for the project plan you designed in Lesson 9.5.
You will then present your findings and assess your work.

Part 1   Collect and Analyze the Data
➤ Collect the data from your chosen population or sample.
➤ Organize the data. Consider using a spreadsheet.
➤ Make an appropriate graph to display your data.
➤ Analyze your results. What conclusions can you make?

Part 2   Assess Your Data
Use the rubric you made to assess your data collection and make
any necessary changes.

Part 3   Present Your Findings
In your presentation, make sure you answer these questions:
➤ Why did you choose your topic?
➤ What considerations did you make when writing your question?
➤ What considerations did you make when selecting the population or sample?
   If you chose a sample, how did you try to ensure the conclusions would be valid?
➤ Why did you choose to display the data in the way you did?
➤ What conclusions can be made from the data?
➤ Are you surprised by the results? Explain.
➤ Who might be interested in your data and why?
➤ If you were to repeat the project, what steps would you take to improve it? Explain.

Part 4   Assess Your Presentation
Use the rubric to assess your presentation.

Your work should include:
• a project plan from Lesson 9.5
• the display of your data and your conclusions
• answers to the presentation questions
• a rubric used for self-assessment

Reflect on Your Learning

What are the most important considerations when
collecting data? Why? What are the most important
questions to ask about probabilities published in the
media? Explain.

A quilt consists of small blocks that tessellate.
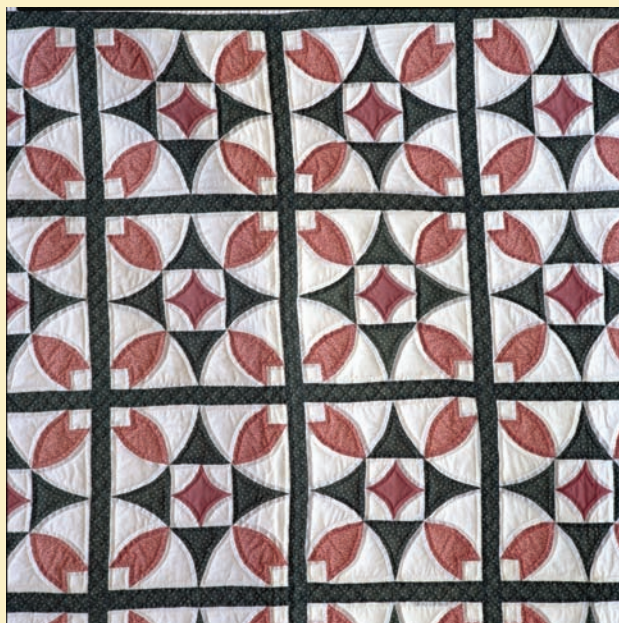
Materials
- ruler
- compass
- pencil crayons or markers
- construction paper
- tape
- dynamic geometry software (*optional*)

Part 1
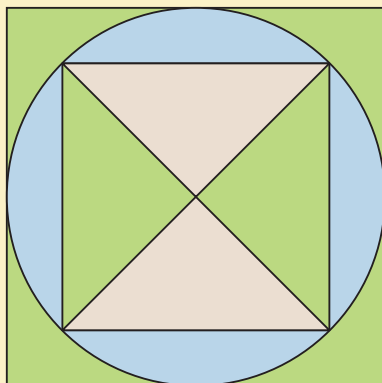
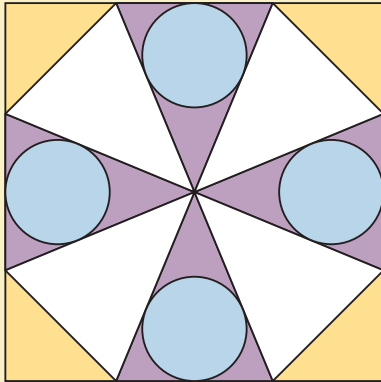The larger shape in this quilt block is a square with side length 15 cm.

Within the square there is a circle and within this circle there is a smaller square.

➤ What is the side length of the smaller square?
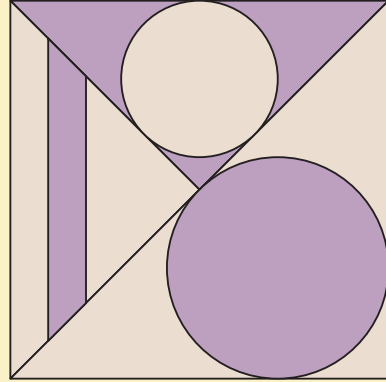
➤ Describe the triangles in the block.

Part 2

Look at these quilt blocks. What shapes do you see?

Block A

Block B

Part 3

Design a square block with side length 15 cm that will be repeated 3 times
to form a square with side length 30 cm. Use circles as part of your design.
With your four 15-cm blocks, create as many different 30-cm blocks as you can.
Describe the symmetries in each 30-cm block.

Take It Further

Choose a shape other than a square.
Create a block that will tessellate to form a quilt.

1. 1. Sketch this number line.

```
+--+--+--+--+--+--+--+--+--+--+
0  1  2  3  4  5  6  7  8  9  10
```
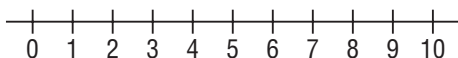
Do *not* use a calculator. Determine or estimate each square root. Where necessary, write the square root to the nearest tenth. Place each square root on the number line.

a) $\sqrt{0.64}$ b) $\sqrt{\dfrac{36}{25}}$ c) $\sqrt{79.7}$ d) $\sqrt{4.41}$

e) $\sqrt{\dfrac{100}{9}}$ f) $\sqrt{\dfrac{89}{90}}$ g) $\sqrt{30.25}$ h) $\sqrt{\dfrac{17}{4}}$

2. Here is a floor plan for a building that is 5 m tall. It has a flat roof. What is the surface area of the building, including its roof, but excluding its floor?

```
         28 m
  ┌──────────────────┐
  │                  │
  │                  │ 15 m
19 m│            ┌───┤
  │            │   │
  │            └─┬─┘
  └──────────────┘
           6 m
```
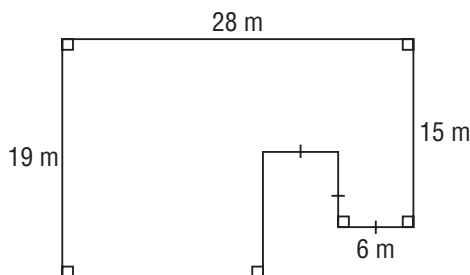
2. 3. A student answered the following skill-testing question to try to win a prize:
$(-4)^3 - (-2)^4 \div 2^2 + 5^2 \times 7^0$

The student's answer was 5. Did the student win the prize? Show your work.

4. Express as a single power, then evaluate.
$\left(\dfrac{6^7 \times 6^3}{6^5 \times 6^2}\right)^2$

3. 5. During the month of July, Bruce earned $225 cutting lawns and $89.25 weeding flower beds. He spent $223.94 on an MP3 player and purchased 3 DVDs at $22.39 each.

a) Write each amount as a rational number. Justify your choice of sign for each number.
b) Write an addition statement for Bruce's balance at the end of July.
c) What is Bruce's balance?

6. Use a calculator. Evaluate to the nearest hundredth.
$$\dfrac{-17.8 - (-9.6) \div 1.2 + 31.4}{7.6 \times (-4.1) - 2.9}$$

4. 7. Marcie is rowing at an average speed of 3 m/s. She travels a distance d metres in t seconds.

a) Create a table of values for this relation.
b) Graph the data. Will you join the points on the graph? Explain.
c) Is the relation linear? How do you know?
d) Write an equation that relates d to t.
e) How far does Marcie row in 15 s?
f) How long does it take Marcie to travel 1 km?

8. Colton works for 8 h each week at a sporting goods store. This graph shows how his pay in dollars relates to the number of weeks he works.

Colton's Pay
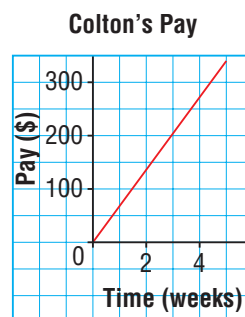
a) Estimate how much Colton earns after 2 weeks.

b) Estimate how long it will take Colton to earn $1000. What assumptions do you make?

c) What conditions could change that would make this graph no longer valid?

5 **9.** The difference of two polynomials is $4n^2 - 2n + 5$. One polynomial is $-6n^2 - 7n + 8$.

a) What is the other polynomial? Show your work.

b) Why are there two possible answers to part a?

10. This diagram shows one rectangle inside another.

$4x + 3$

$3x$ $2x$ $3x + 1$
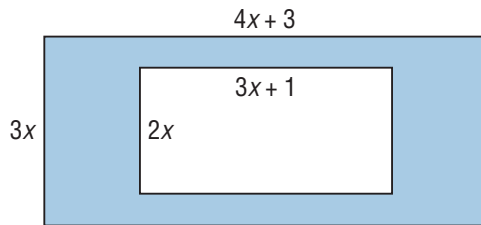
a) Determine the area of the shaded region. Justify your answer.

b) Determine the area of the shaded region when $x = 1.5$ cm.

6 **11.** Mountain bikes can be rented from two stores near the entrance to Stanley Park. Store A charges $6.00 per hour, plus $3.50 for a helmet and lock. Store B charges $6.70 per hour and provides a helmet and lock free. Determine the time in hours for which the rental charges in both stores are equal.

a) Write an equation to solve this problem.

b) Solve the equation.

c) Verify the solution.

12. Jerry hires a pedicab to tour a city. He is charged $2.75 plus $0.60 per minute of travel. He has $12.00. How long can he ride in the pedicab?

a) Choose a variable and write an inequality to solve this problem.

b) Solve the inequality. Explain the solution in words.

c) Verify the solution.

d) Graph the solution.

7 **13.** This photo is to be enlarged. Determine the dimensions of an enlargement with each scale factor.

**a)** 2   **b)** $\frac{7}{4}$ **c)** 3.5

14. A hockey rink measures 60 m by 26 m. A model of a hockey rink measures 1.5 m by 0.65 m.

a) What is the scale factor for this reduction?

b) A hockey goal is 1.8 m high and 1.2 m high. What are the dimensions of a goal on the model hockey rink?

15. Bobbi wants to determine the height of a building. When Bobbi's shadow is 2.5 m long, the shadow of the building is 12 m long. Bobbi is 1.7 m tall. What is the height of the building, to the nearest tenth of a metre? Show your work.

16. Trapezoid ABCD is part of a larger shape.
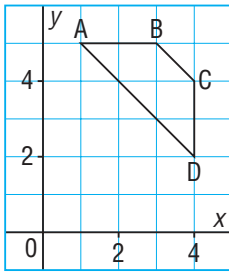

After each reflection below:
- Draw the image of ABCD.
- Describe any symmetry in the shape and its image.

a) a reflection in the horizontal line through 5 on the y-axis

b) a reflection in the vertical line through 4 on the x-axis

c) a reflection in the oblique line through $(0, 6)$ and $(6, 0)$

**17. a)** Does rectangle MNPQ below have rotational symmetry about its centre? If it does, state the order and the angle of rotation symmetry.
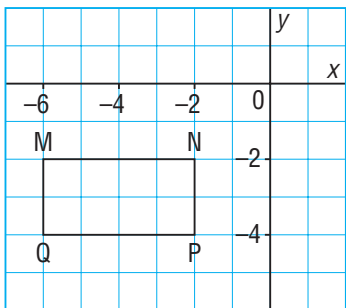
b) Rectangle MNPQ is part of a larger shape. It is to be completed in three different ways, by each rotation below:
- 90° clockwise about the point $(-2, -3)$
- 180° about vertex Q
- 270° clockwise about the point $(-4, -4)$

i) Draw each rotation image.

ii) List the coordinates of the larger shape formed by the rectangle and its image each time. Describe any rotational symmetry in this shape.
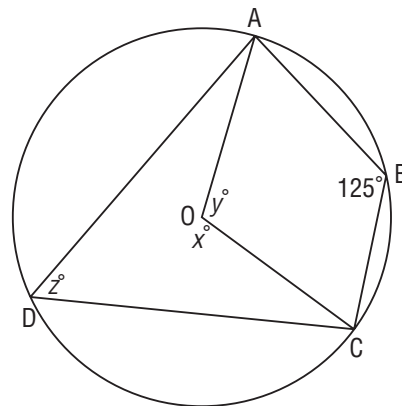
8 **18.** Point G is a point of tangency and O is the centre of the circle. Determine the length of GH to the nearest tenth of a centimetre.
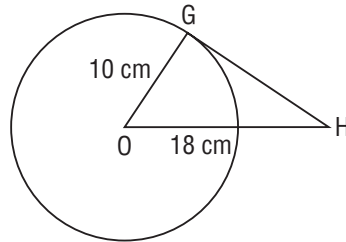

19. A circle has diameter 27 cm. How far from the centre of this circle is a chord 18 cm long? Give your answer to the nearest tenth of a centimetre.
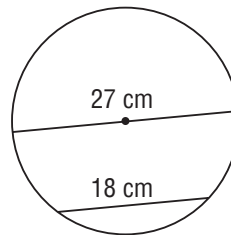

20. Point O is the centre of a circle. Determine the values of $x°$, $y°$, and $z°$.

21. A rectangle is inscribed in a circle with radius 14 cm. The length of the rectangle is 21 cm. Determine the width of the rectangle to the nearest tenth of a centimetre.

22. A regular decagon is inscribed in a circle with radius 20 cm and centre O. The distance from O to each side of the decagon is about 19 cm. What is the perimeter of the decagon to the nearest centimetre?
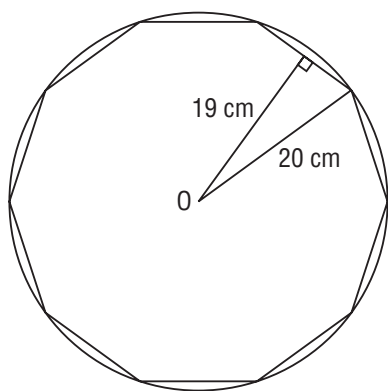
23. Point O is the centre of a circle. Determine the values of $x°$, $y°$, and $z°$.
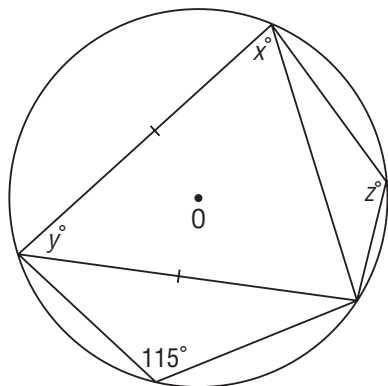
9 24. A baseball team won 58 of its first 100 games of the season. Bao concludes that there is a 58% probability of the team winning its next game.
a) What assumptions is Bao making?
b) For each assumption, explain how the probability might change if the assumption is not true.

25. Zahara is planning a telephone survey to discover how much weekly allowance parents give their children.
a) Identify potential problems she may encounter related to 3 of these factors: bias, timing, privacy, cultural sensitivity, ethics, time
b) For each potential problem in part a, explain how Zahara could avoid the problem.

26. An on-line fashion magazine for teens concludes that high school students spend on average $200 per month on clothes.
a) How do you think the magazine may have conducted the survey?
b) Do you think the conclusion is valid? Explain.

27. For each situation, explain why data are collected from a sample and not a census.
a) to determine the mean cost of hockey equipment for teenagers in Canada
b) to determine the number of Canadian families with at least one cell phone

28. Should a census or sample be used to collect data about each topic? Explain your choice.
a) to determine the popularity of a new television show
b) to determine the condition of an airplane's seatbelts

29. Discuss whether each sampling method would lead to valid conclusions.
a) To determine if the prices of items in a grocery store are appropriate, you survey every 12th customer leaving the store on a given day.
b) To determine the favourite video game of students in a school, you survey 20 randomly selected students from each grade in the school.