The Language of Mathematics



After this lesson, you will be able to...

5.1

- use mathematical terminology to describe polynomials
- create a model for a given polynomial expression



The Great Wall is the world's largest human-made structure. It stretches over 6700 km across China. The wall was created by joining several regional walls.

Similarly, mathematics is a developing science made up of several branches, including arithmetic, geometry, and **algebra**. It is a science that studies quantity, shape, and arrangement. As with any science, mathematics comes with its own unique language. The language of mathematics is universal. It can be understood anywhere in the world.

Look at the following paragraph. How much of it can you read? What languages do you think the paragraph contains?

Would the algebraic equations be any different if the paragraph was written in any other modern language?

algebra

 a branch of mathematics that uses symbols to represent unknown numbers or quantities

Explore the Language of Algebra

- a) For the algebraic expression 5a + 4b, what terminology can you use to describe the numbers 4 and 5, and the letters a and b?
 - **b)** What terminology can you use to describe the expression $-7x^2$ and its parts?
- **2.** Make up a real-life situation and write an algebraic expression for it. What do the parts of your expression represent?
- 3. Algebraic expressions can have different numbers of terms.

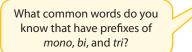
Number of Terms	Examples
	5
1	7x
1	-3ab
	$\frac{y}{2}$
2	$5 + x$ $3x^2 - 2$
2	$ \frac{3x - 2}{7xy + z^2} $
	1-x+y
3	$2x^2 - 3x + 5$
	a + b + c



- an expression formed from the product of numbers and/or variables
- 9x is a term representing the product of 9 (coefficient) and x (variable)
- a constant value, such as 5, is also a term

Write other examples of expressions with one term, two terms, and three terms.

4. A study of algebra includes working with **polynomials**. They are named by their number of terms. How do the names *monomial*, *binomial*, and *trinomial* relate to the number of terms in an expression?



Reflect and Check

5. Look at the algebraic equations in the paragraph written in Chinese on the previous page. Use as much algebraic terminology as you can to describe them.

🕥 Language Link

The word *algebra* comes from Arabic. The word originated in Iraq. Around A.D. 830, Mohammad al-Khwarizmi of Baghdad wrote a book called *Hisab al-jabr w'al-muqabalah*. This book summarized Hindu understandings of equations and how to solve them. The whole title was too hard for some Europeans so they kept only the word *al-jabr*. We get the term *algebra* from that Arabic word.

polynomial

- an algebraic expression made up of terms connected by the operations of addition or subtraction
- 3x² 4 has two terms.
 3x² and 4 are connected by the operation of subtraction.



For more information about the history of algebra, go to www. mathlinks9.ca and follow the links.

OD Literacy Link

Types of Polynomials

Some polynomials have specific names. *Monomial*: one term *Binomial*: two terms *Trinomial*: three terms

All expressions with one or more terms are called *polynomials*. *Polynomial* means many terms.

Link the Ideas

Example 1: Name Polynomials by the Number of Terms

For each expression, identify the number of terms and whether it is a monomial, binomial, trinomial, or polynomial.

a) $4xy + 3$	b) $7a^2 - 2ab + b^2$
c) $5x^2 + y^2 + z^2 - x - 6$	d) 13

Solution

Expression	Number of Terms	Name	
a) $4xy + 3$	2	binomial	What are the
b) $7a^2 - 2ab + b^2$	3	trinomial	two terms in $4xy + 3?$
c) $5x^2 + y^2 + z^2 - x - 6$	5	polynomial	
d) 13	1	monomial	

Show You Know

For each expression, identify the number of terms and whether the expression is a monomial, binomial, trinomial, or polynomial.

a) 5j ²	b) $3 - m^2$
c) $ab^2 - ab + 1$	d) $-4x^2 + xy - y^2 + 10$

degree of a term

- the sum of the exponents on the variables in a single term (monomial)
- 3xz, or $3x^{1}z^{1}$, has degree 2, since 1 + 1 = 2
- 5x²y and −2b³ are terms of degree 3

degree of a polynomial

- the degree of the highest-degree term in a polynomial
- In 7a² 3a, the degree of the first term is 2, and the degree of the second term is 1. The highest degree is 2, so the degree of the polynomial is 2.

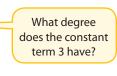
Example 2: Identify the Number of Terms and Degree of a Polynomial

What is the number of terms and the degree of each polynomial?

a) $4x^2 + 3$	b) $7a^2 - 2ab + b^2$
c) $5x + z - 6$	d) 7

Solution

a) The polynomial 4x² + 3 has two terms. The term with the highest degree is 4x². Its degree is 2. The degree of the polynomial is 2.



b) The polynomial $7a^2 - 2ab + b^2$ has three terms. Each of the three terms has a degree of 2. The degree of the polynomial is 2.

When a variable has no exponent, the value of the exponent is 1. How do you know 2*ab* has a degree of 2?

- c) The polynomial 5x + z 6 has three terms. Both of the terms 5x and z have a degree of 1. The degree of the polynomial is 1.
- d) The number 7 is one term that is constant. A constant term has a degree of 0. The degree of the polynomial is 0.

Show You Know

What is the number of terms and the degree of each polynomial? Explain each answer.

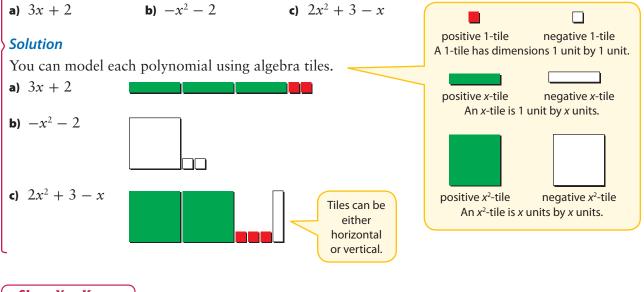
a) $1 - 3x$	b) $4x - 3xy + 7$
c) $-27b^2$	d) 99

OD Literacy Link

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When a term has more
than one variable, the
variables are usually
written in alphabetical
order.
Examples: 5ab, -12x^2y^2
```

Example 3: Model Polynomials

Model each polynomial.



Show You Know

- **a)** Model $-x^2 + 4x 3$.
- **b**) What expression is shown by the algebra tile model?



For practise using virtual algebra tiles, go to www.mathlinks9.ca and follow the links.

Key Ideas

- Algebra is a branch of mathematics that uses symbols to represent unknown numbers or quantities. The symbols are often letters and are called variables.
- Polynomials are made up of terms. Terms are connected by addition or subtraction.

 $3x^2 + 2x - 7$ has three terms.

• Polynomials can have one or more terms. Some polynomials have specific names.

Name	Number of Terms	Example
monomial	1	$6x^2$
binomial	2	$3a^2 - 5$
trinomial	3	$-w^2 + 5w + 1$
polynomial	more than 3	$2s^2 - t^2 + st + 7t - 4$

• Each algebraic term has a degree. You can find the degree of a term by adding the exponents of the variable(s) in the term.

3x has degree 1. $-5x^2y$ has degree 3.

• A polynomial has the same degree as its highest-degree term.

```
x^{2} + 5x - 7 has degree 2.
```

12 has degree 0.

• You can use models, such as algebra tiles and diagrams, to represent some polynomials.

 $2x^2 - 4x + 3$



Check Your Understanding

Communicate the Ideas

- 1. Identify at least three mathematical words or phrases you could use to refer to the polynomial $-5 + x^2$.
- 2. Sonja and Myron are discussing the algebra tile model shown below.



Sonja says, "The model represents the expression $3x^2 + x + 2$." Myron claims, "It represents $3x^2 - x - 2$." Who is correct? How do you know?

- 3. Give two examples of a polynomial that satisfies all statements.
 - consists of two terms
 - contains two variables
 - has degree 2
 - one term is of degree 1 and has a coefficient of 1
- **4.** When is it acceptable not to write the 1 in an algebraic expression? When must you write the 1? Give examples.

Practise

For help with #5 to #7, refer to Example 1 on page 176.

5. For each expression, identify the number of terms and whether the expression is a monomial, binomial, trinomial, or polynomial.

a)
$$3x^2 - 5x - 7$$
 b) $-11a$
c) $c^2 + cf + df - f^2$ d) 8

- **6.** What is the number of terms and what is a name for each expression?
 - a) n b) $6 + 4x x^2$
 - c) 0 d) $p^2 + 3pq$
- **7.** Refer to the polynomials below to answer each question.

6 <i>x</i>	-15
3x - y	$4c^2 - cd$
7 + a + b	$3m^2 - 4mn - 9n^2 + 1$

- a) Which ones are monomials?
- **b)** Which ones are trinomials?
- c) Which ones have two terms?

For help with #8 to #10, refer to Example 2 on pages 176–177.

8. For each polynomial, what is the degree and number of terms?

a)
$$4 - b$$

b) $fg + 2g$
c) $8x^2 - xy - y^2$

- **9.** State the degree and number of terms for each polynomial.
 - a) 3xy + 1 b) $11k^2 + 7k 5$

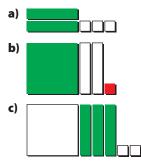
10. Refer to the polynomials below to answer each question.

$3b^{2}$	2 + p
4st + t - 1	$2x^2 - y^2$

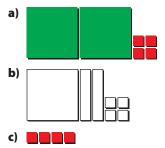
- a) Which ones are binomials?
- **b)** Which ones have degree 2?
- c) What is the variable in the monomial?
- d) Which polynomials have a constant term?

For help with #11 to #14, refer to Example 3 on page 177.

11. What expression is represented by each set of algebra tiles?



12. Write the expression represented by each set of algebra tiles.



13. Model each polynomial.

- a) $x^2 + x 1$ b) 3x + 2
- c) -2x
- **14.** Use a model to represent each polynomial.
 - a) $-x^2 + 3$ b) $2x^2 - 3x$
 - **c)** 8

Apply

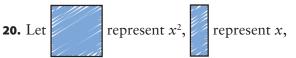
- **15.** Represent each of the following with a diagram and an expression.
 - a) binomial
 - **b**) monomial of degree 1
 - c) monomial of degree 2 with a coefficient of 9
 - **d)** polynomial with four terms that is of degree 2
- **16.** Use your knowledge of algebra tiles to answer the following questions.
 - a) How are the dimensions of a 1-tile and an *x*-tile related?
 - b) The rectangle shown was formed using an x²-tile and three x-tiles. What is an expression for the length of the rectangle?



- **17.** Write an algebraic expression for each of the following.
 - **a)** the product of 6 and x
 - **b)** the sum of 2x and 3
 - c) the length of the rectangle below, which is made from algebra tiles



- 18. Make a model of an algebraic expression that includes at least one x²-tile, at least two x-tiles, and two 1-tiles. Use materials or a diagram. Then, use symbols to show your expression. What type of polynomial is it?
- **19.** For the polynomial $6x^2 5$, state the following.
 - a) number of terms
 - **b**) coefficient of the first term
 - c) number of variables
 - d) degree of polynomial
 - e) constant term



and *prepresent 1*. The same diagrams in yellow represent negative quantities.

a) What is an expression for the polynomial shown?



- **b)** Make up a trinomial. Draw diagrams to represent your trinomial.
- **21.** Write each statement as an algebraic expression. Include what your variables represent.
 - a) Eight and a number are added together.
 - **b)** Omar has some money in his wallet. How much money does he have after a friend gives him \$5?
 - c) A page is 4 cm longer than its width.
 - **d)** The product of a number and 5 is increased by 2.
 - e) The result of 3 times the number of people decreased by 21.
- **22.** Describe a situation that could be modelled by each given polynomial.
 - **a)** 3x + 5 **b)** 10 x

- **23.** Marion gives French lessons in the evening. She charges \$20 for adults and \$15 for children. The expression 20a + 15crepresents her earnings.
 - a) What do the variables *a* and *c* represent?
 - b) How much does Marion make if she gives lessons to four adults and nine children? Show your work.
 - c) Write a new expression for Marion's earnings if she charges \$3 more for adults and \$2 more for children.
- **24.** Tickets for a school concert are \$10 for adults and \$5 for students. Write an expression that shows the total income for the school concert. Tell what your variables represent.
- **25.** A hockey league awards teams two points for a win, one point for a shoot-out loss, and no points for a loss in regulation time.



- a) Write an algebraic expression to represent the total points for a hockey team.
- **b)** What variable(s) did you use? Indicate what each variable represents.
- c) In the first 20 games of the season, Team A had 12 wins and 4 shoot-out losses. How many losses in regulation time did the team have?
- d) What were the total points for Team A?
- e) Team A was tied with Team B after 20 games. However, Team B had a different record than Team A. Show two possible records for Team B. Use your expression to show that the two hockey teams had the same number of total points.

- **26.** A banquet hall can be rented for parties. An expression for the rental cost is 5n + 75, where *n* is the number of people.
 - a) What type of polynomial is 5n + 75, and what is its degree?
 - **b**) What could the numbers 5 and 75 represent?
 - c) How much does it cost to rent the banquet hall for 150 people?

Extend

- 27. On a true/false test, there is a penalty for incorrect answers. Miranda's teacher advises the students not to guess at any of the 25 questions. The teacher awards 2 points for a correct answer, -1 point for a wrong answer, and 0 points if the question is not answered.
 - a) Write a polynomial to represent a student's score on this test.
 - **b)** What are the maximum and minimum scores possible on this test? Explain.
 - c) What are all of the possible scores if Miranda got 20 questions correct? Explain.
- 28. What is the degree of xy - abx + cdy - qr - prqz if x, y, and z are variables and a, b, c, d, p, q, and r are coefficients?
- **29.** Ricardo draws the following rectangle with dimensions in metres.



- a) What is an expression for the perimeter of the rectangle?
- **b)** Write an equation showing how the length and width of the diagram would be related if the dimensions given were for a square.
- c) Solve your equation in part b) to find the value of *x*. Show your work.

- **30.** Create a polynomial satisfying the following conditions:
 - contains three variables
 - has three terms
 - is of degree 2
 - has a constant term, 3
- 31. Deidra is training for a triathlon. From her training, she knows that she can swim at 1.3 km/h, cycle at 28 km/h, and run at 12 km/h.
 - a) Write the formula distance (d) = speed × time using variables of your choice for speed and time. Tell what each variable represents.

The distance for each leg of the competition is different. In the table, this difference is shown using subscripts.

b) Use your formula to complete the table.

Part of Distance Race (km)		Speed (km/h)	Time (h)	
swim	$d_{_1}$	1.3	$\frac{d_1}{1.3}$	
cycle	d_2	28.0		
run	d_{3}	12.0		

- **c)** Write a trinomial to model Deidra's total time for a triathlon.
- d) A triathlon includes a 1.5-km swim, a 40-km cycle, and a 10-km run. How long will it take Deidra to complete this triathlon? What assumptions are you making?
- e) If Deidra could maintain the same speeds, how long would it take her to complete a triathlon that is a 3.8-km swim, 180-km cycle, and 42.2-km run?

WWW Web Link

The Ironman Canada Triathlon in Penticton, B.C., involves a 3.8-km swim, 180-km cycle, and 42.2-km run. The times of recent winners are impressive. Top men's times are around 8 h, while women's times are around 9 h. For information about the Ironman Canada Triathlon and about the history of the Ironman competition, go to www. mathlinks9.ca and follow the links.

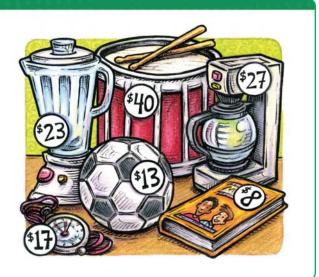


Math Link

You want to be a contestant on a game show. In order to get on the show, you must show how to spend exactly \$100 by choosing from the items shown.

You may purchase some or all of the six items, and as many of a single item as necessary.

- a) Find at least six answers that would get you on the game show.
- **b)** Write an algebraic expression for one of your combinations in part a). What is an equation for this same combination?
- c) Is it possible to spend \$100 choosing all different items? Explain.



5.2

Equivalent Expressions

Focus on...

After this lesson, you will be able to...

- use algebra tiles and diagrams to show whether expressions are equivalent
- identify equivalent expressions that are polynomials
- combine like terms in algebraic expressions

Materials

• concrete materials, such as algebra tiles

Today's space program requires extensive use of algebra. Computer programs control shuttle flights and manipulate the Canadarm. They also control conditions inside the International Space Station. These programs use algebraic models, expressions, and equations. Where else in the real world is algebra used?

Explore Combining Like Terms

The astronauts on the space shuttle have a limited amount of living space. They eat, sleep, and relax in a rectangular space with a width of only about 3.2 m and a length that is 0.8 m greater than the width.

1. What is the length of the living space? How do you know?



- **2.** Draw and label a diagram of the rectangular living space. Find the perimeter of the rectangle. How did you find the perimeter?
- **3.** Draw another rectangle. The length is still 0.8 m greater than the width but you have no known value for the width. What could you use to represent the width of the rectangle? What would be an expression for the length of this rectangle?
- **4.** Write an expression for the perimeter of the second rectangle.
- 5. How many terms are in your expression for the perimeter?
- 6. Use materials or a diagram to model your expression for the perimeter.
- **7.** Rearrange your model so similar objects, shapes, or variables are all together. Combine the similar objects, shapes, or variables. What is an equivalent expression for the perimeter?

Reflect and Check

- **8.** What do you think *like terms* means? Give examples to support your ideas.
- 9. How do you combine like terms in polynomials? Explain with examples.

Link the Ideas

Example 1: Identify Coefficients, Variables, and Exponents

For each expression, identify the coefficient, the variable(s), and the exponent of each variable.

a)	3 <i>w</i>	b) <i>a</i> ²	c) $-4xy$	d) -g
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Solution

Show You Know

Expression	Coefficient	Variable(s)	Exponents of the Variable(s)
a) 3w	3	w	1
b) <i>a</i> ²	1	а	2
c) $-4xy$	-4	<i>x</i> and <i>y</i>	1 and 1
d) -g	-1	g	1

Give the coefficient, the variable(s), and the exponent of each variable.

c) *b*

d) $7st^2$

OD Literacy Link

Even if a term has two variables, it always has only one coefficient. Example: A term would be written as -6xy, not as -2x3y.

like terms

- terms that differ only by their numerical coefficients
- examples of like
 terms are
 - 3x and -2x
 - 6y² and −4y²
 −5xy and yx
 - - 5xy and y • 17 and - 8

Example 2: Identify Like Terms

b) -x

Identify the like terms in each group.

a) $5b^2$	3 <i>cb</i>	-2b	7 <i>c</i>	6 <i>b</i>
a) $5b^2$ b) $3x^2$	4xy	$-2x^{2}$	$7x^{2}$	$\frac{1}{2}y$
c) 3pq	11	$-4q^{2}$	-3	pq

Solution

a) $3c^2$

- a) -2b and 6b are like terms. Both have a variable b with an exponent of 1. All the other terms are unlike.
- **b**) $3x^2$, $-2x^2$, and $7x^2$ are like terms. Each of them has a variable x with an exponent of 2. The other terms are unlike.
- c) 3pq and pq are like terms. Both have variables p and q, each with an exponent of 1. The terms 11 and -3 are also like terms.

Show You Know

- a) Give an example of three like terms.
- **b)** Identify the like terms in the following group: $6t \quad 3s \quad 6t^2 \quad 6st \quad -8s$

Example 3: Combine Like Terms

Combine like terms in each expression.

a) 4x - 2x + 3 - 6b) $2x^2 + 3x - 1 + x^2 - 4x - 2$ c) $4 - x^2 + 2x - 5 + 3x^2 - 2x$

Solution

Method 1: Use a Model

a) 4x - 2x + 3 - 6

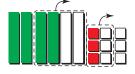
You can use algebra tiles to represent each term.



Group the tiles to form zero pairs and remove the pairs.

Like terms can be combined

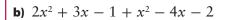
to simplify expressions.



Write an expression for the remaining tiles. 2x - 3

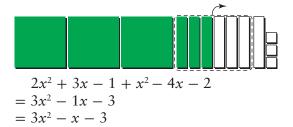


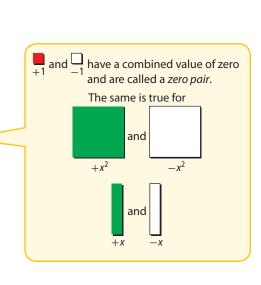
So, 4x - 2x + 3 - 6 = 2x - 3.





Group like terms and remove the zero pairs.





Strategies

Model It

OD Literacy Link
Any term with a variable having a coefficient of 1 can be written without its numerical coefficient.
However, the sign must remain. Example:
-1x = -x +1x = x $-1x^{2} = -x^{2}$ +1x ² = x ²
Coefficients without a sign are positive.

c) $4 - x^2 + 2x - 5 + 3x^2 - 2x$ Group like terms and remove the zero pairs. $4 - x^2 + 2x - 5 + 3x^2 - 2x = 2x^2 - 1$

Method 2: Use Symbols

- a) Add or subtract the coefficients of like terms. 4x - 2x + 3 - 6 = 2x - 3
- **b)** A polynomial with more than one term can be written in different orders. Rearrange by grouping like terms.

 $2x^{2} + 3x - 1 + x^{2} - 4x - 2$ = 2x² + x² + 3x - 4x - 1 - 2 = 3x² - x - 3

c) $4 - x^2 + 2x - 5 + 3x^2 - 2x$ = $-x^2 + 3x^2 + 2x - 2x + 4 - 5$ = $2x^2 - 1$

Show You Know

Combine like terms. a) $5x - 3x^2 + 2x - x^2$ b) 2x - 6 - 2x + 1c) $k - 2k^2 + 3 + 5k^2 - 3k - 4$

Key Ideas

• An algebraic expression is made up of terms. Each term can have any number of variables. Each variable has an exponent. A constant term, such as 9, has no variable.

• Like terms differ only by their numerical coefficients.

Like terms can be combined.

- Like terms:
- -7x and 3x
- w^2 , $3w^2$, and $0.5w^2$
- 6 and 15

Term	Coefficient	Variable(s)	Variable's Exponent
6 <i>p</i> ²	6	Þ	2
$-x^2y$	-1	<i>x</i> , <i>y</i>	2 for <i>x</i> , 1 for <i>y</i>

- Unlike terms cannot be combined.
- Unlike terms:
- $6x \text{ and } 3x^2$
- m^2n and $4mn^2$
- 7 and 7*d*

🕐 Literacy Link

In algebra, terms are often arranged in descending order by degree. For example, $-3y + 4y^2 - 1$ is written as $+4y^2 - 3y - 1$ or $4y^2 - 3y - 1$. This makes it easier to compare expressions. Answers are usually written this way.

WWW Web Link

To learn more about the parts of a polynomial and combining like terms, go to www. mathlinks9.ca and follow the links.

Check Your Understanding

Communicate the Ideas

- **1.** Using models, show how you know that s 5s combines to give -4s.
- **2.** Jean claimed that $3m^2 + 4m$ could be combined to give $7m^3$. Do you agree? Explain with diagrams.
- **3.** Most people would agree that 2 cats + 5 cats = 7 cats and 7 cats 2 cats = 5 cats. Use this information to support an argument for combining like terms and for being unable to combine unlike terms in algebra. Use examples with two different animals and two different variables.
- **4.** Does a number in front of a variable affect whether or not you have like terms? Explain using examples.

Practise

For help with #5 to #7, refer to Example 1 on page 184.

- 5. What is the value of the coefficient and the number of variables for each term?
 a) -3z²
 b) k
 c) 43
- **6.** Determine the value of the coefficient and the number of variables for each term.
 - **a)** 4d **b)** -prt **c)** $-8fg^2$
- **7.** Use the following monomial expressions to answer the questions below:

3 <i>x</i>	4 <i>t</i>	x^2
-ts	xt	$2t^{2}$

- a) Which have a coefficient of 1?
- **b)** Which have two variables?
- c) Which have only one variable with an exponent of 1?
- d) Which have a coefficient of -1?

For help with #8 and #9, refer to Example 2 on page 184.

- **8.** Identify the like terms in each group.
 - a) 2a 5 -7.1a 9b -c b) 3m -2ab $\frac{4}{3}m$ -2ad m^2 c) -1.9 $6p^2$ 5 -2p p^2

- **9.** Which terms are like terms in each group?
 - a) -2k 9 104k 104f $-f^2$ b) $\frac{1}{2}ab$ 0.5a -4b ab^2 abc) -5 $13d^2$ 5 -10d d^2

For help with #10 to #12, refer to Example 3 on page 185.

10. Collect like terms.

a) $3x - 2x^2 + x - 2x^2$ b) $-4 - 2n^2 - 3n + 3 + 2n^2$ c) $2q - 4q^2 - 2 + 3q^2 + 2 - 3q$ d) -4c + 3 + 5c - 7e) $h^2 - 3h + 4h^2 + 2h$ f) $3j - 5 + 2j^2 - 1 + 2j - 3j^2$

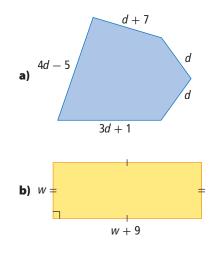
11. Simplify by collecting like terms.

a) $2d - 3d^2 + d^2 - 5d$ b) $y^2 + 2y - 2y^2 + y$ c) $-p + 4p^2 + 3 - 3p^2 - 5 + 2p$ d) m - 4 + 6 + 3me) $q^2 - 3q + 2q^2 - q$ f) $5w - 3 + w^2 - 2w - 4w^2 - 1$

- **12.** Which expressions are equivalent to the simplified expression $-3x^2 + x 4$?
 - **A** $-4 + 3x^2 + x$ **B** $x - 4 - 3x^2$ **C** $x^2 + 2 - 4x^2 + 3x - 6 - 2x$ **D** $-3 - 5x^2 + x + 1 + 2x^2$ **E** $2x - 2 + x^2 - x - 4x^2 - 2$ **F** $-4 - 3x - 3x^2 - 0 + 5x^2 + 4x - 6x^2$

Apply

- **13.** Jessica and Taz are working on a measurement problem. Their calculations involve combining Jessica's measurement of 2 m and 32 cm with Taz's measurement of 1 m and 63 cm. Jessica claims you find the answer just like in algebra. Do you agree? Explain.
- **14.** Describe a real-life situation that could be represented by each expression.
 - **a)** m-3 **b)** 2p+5
- **15.** For each of the following polynomials, write an equivalent expression with six terms.
 - a) $2p^2 3p + 2$ b) $-3x^2 + 5x - 4$ c) $4r^2 - 2q^2 - 3qr$
- **16.** Write an expression for the perimeter of each figure. Then, combine like terms if possible.



- **17. a)** Draw a figure with a perimeter that is represented by (5s) + (3s 2) + (s + 6), where each value in parentheses represents the length of one side.
 - **b)** Simplify the expression for the perimeter.
- 18. A student council decides to raise money by organizing a dance. The cost of a band is \$700. The student council decides to sell tickets at \$5 each.
 - a) Write an expression for the profit that the student council would make. What does your variable represent?
 - **b)** If 250 students pay to attend the dance, what is the profit?
 - c) Estimate, then calculate, the minimum number of students who will need to buy tickets for the student council to make a profit.
- 19. A heating company charges \$60 per hour plus \$54 for a service call. Let *n* be the number of hours the technician works at your house.
 - a) What expression represents the total cost of the job?
 - **b)** What is the cost for a job requiring 2 h?
 - c) The company charges half as much for a second technician. Write an expression showing the total cost if two technicians install a new furnace. Simplify your expression by combining like terms.
- **20.** A publisher makes books for a number of distributors. For one book, the charge to the distributor is represented by a fixed cost of \$3000 plus \$16 per book.
 - a) Write an expression for the cost that a distributor is charged for *b* copies of this book.
 - **b)** How much do 600 books cost?
 - **c)** What is the cost per book if 600 are ordered?
 - **d)** What is the cost per book if 1000 are ordered?

- **21.** Raj was told to write an expression equivalent to 3x 8 5x + 9.
 - 3x 8 5x + 9= 3x - 5x - 8 + 9
 - = 2x 1
 - a) What errors did he make?
 - **b)** Show the correct response.
- 22. The diagram represents a piece of string.

x = 3x + 7 = 2x - 5

- a) What is an expression for the total length of the string?
- **b)** Combine like terms to get the simplest expression possible for the length of the string.

Extend

23. When would the expressions x + y + 3 and x + w + 3 be equal? How do you know?

- **24.** A department store marks up wholesale prices 40% to get its retail or selling price.
 - a) Complete the following table. The first row has been done for you.

Wholesale Price (\$)	Expression For Retail Price	Retail Price (\$)	
8.00	8 + (0.4)(8)	11.20	
12.00			
30.00			
x			

- b) How could you find the retail price if the wholesale price is x + 10 dollars? Show two ways to find the answer.
- 25. Zip Publishers will print posters for fundraising events for an initial cost of \$100 plus \$2 per poster. Henry's Printers charges \$150 plus \$1 per poster.
 - a) Write an expression for each company, showing the total cost for any number of posters.
 - **b)** What is the cost of 125 posters from each company?
 - c) What is the total cost if you print 200 posters at each company? Show two different ways to find the answer.

Math Link

Refer to the Math Link for section 5.1 on page 182. Represent each item with a variable:

a = blender	b = watch	c = book
d = soccer ball	e = drum	f = coffeemaker

- a) Rewrite all your combinations that add to \$100, using the letters *a* to *f*. Arrange each combination in alphabetical order. For example, a + e + d + 3c would be written as a + 3c + d + e.
- b) The example in part a) can be used to find other combinations. Notice that e has a value of \$40. What other items from the list have a value of \$40? By substituting into e the letter or letters that combine to a total value of \$40, you arrive at another answer. Do not forget to combine like terms and arrange each expression in alphabetical order. What other combinations can you find using substitution?
- c) If you were asked to find combinations of the items that add to \$101, how could you use algebra to help you? Give two ways that algebra could help you.



Adding and Subtracting Polynomials

Focus on...

After this lesson, you will be able to...

- add polynomial expressions
- subtract polynomial expressions
- solve problems using the addition and subtraction of polynomials

Use the same variable that you used in #1. Why might you be able to use the same variable?

A music store rents out a drum kit for \$55 per month, plus a deposit of \$30. Is there a pattern? How could you use a polynomial expression to represent this pattern?

Explore Adding or Subtracting Polynomial Expressions



- 1. In first semester, Kira decides to play drums for music class. To rent a drum kit, it costs \$55 per month, plus a \$30 deposit.
 - a) What is the total cost of renting the drum kit for three months, including the deposit?
 - **b)** Write an expression to show the total cost for any number of months, including the deposit. Tell what your variable represents.
- **2.** In second semester, Kira decides to play electric guitar. To rent an electric guitar, it costs \$22 per month, plus a \$20 deposit.
 - a) What is the total cost of renting an electric guitar for three months, including the deposit?
 - **b**) Write an expression to show the total cost for any number of months, including the deposit.
- **3.** Mark wants to learn to play both drums and electric guitar. What is an expression for the total cost of renting a drum kit and a guitar for any number of months, including deposits? Then, show how to find a simpler expression.
- **4.** What is an expression for the difference between the cost of renting a drum kit and the cost of renting a guitar for any number of months, including deposits? Then, show how to find a simpler expression.

Reflect and Check

- **5.** a) Describe how to add or subtract polynomial expressions.b) Why was it necessary to use the same variable for each expression?
- **6.** Make up your own situation that involves the rental of two items. Write an expression for the total cost and for the difference in cost.

Link the Ideas

Example 1: Add Polynomials

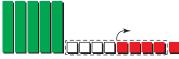
Add 3x - 4 and 2x + 5. Simplify your answer by combining like terms.

Solution

Method 1: Use a Model You can use algebra tiles to model each polynomial.



Arrange the model so that like objects are together. Remove zero pairs if necessary.



Rewrite the model in simplest form.



$$(3x - 4) + (2x + 5) = 5x + 1$$

Method 2: Use Symbols You can combine like terms. (3x - 4) + (2x + 5)= 3x + 2x - 4 + 5= 5x + 1

Show You Know

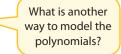
Use two methods to show each addition of polynomials. Give your answers in simplest form.

a)
$$(2a - 1) + (6 - 4a)$$

b)
$$(3t^2 - 5t) + (t^2 + 2t + 1)$$



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Strategies Model It

Example 2: Determine Opposite Expressions

What is the opposite for each of the following?

a) 3*x* c) 4x - 1

b)
$$-2$$

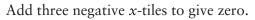
d) $a^2 - 3a + 2$

Solution

a) You can use algebra tiles to represent 3x.

How might you use a diagram to model 3x?







The opposite of 3x is -3x.

You can also use inspection. 3x is positive. The opposite of positive is negative. The opposite of 3x is -3x.

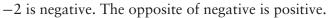
b) You can use algebra tiles to represent -2.

Add two positive 1-tiles to give zero.



The opposite of -2 is +2 or 2.

You can also use inspection.

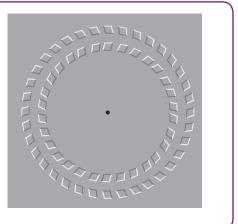


The opposite of -2 is +2 or 2.

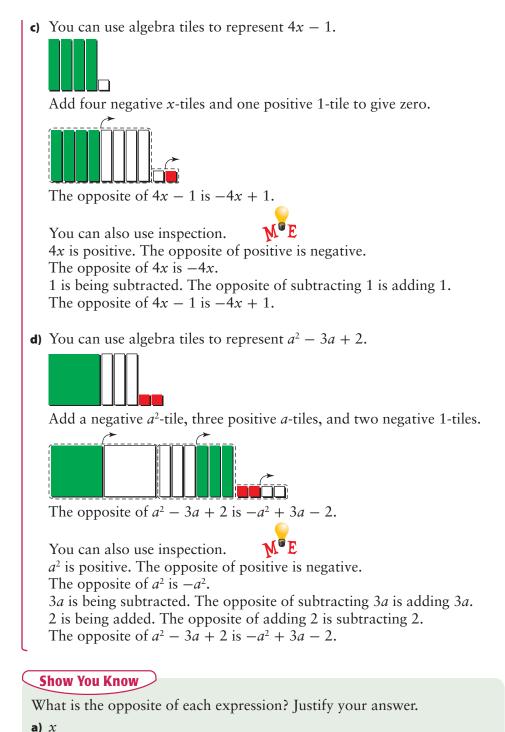
O Art Link

Opposites can be used to create interesting optical illusions.

Look at the dot in the center of the rings. Lean forward and backward. The rings appear to rotate in opposite directions.







b) 5 - 3x

c) $7x^2 + 5x - 1$

Example 3: Subtract Polynomials

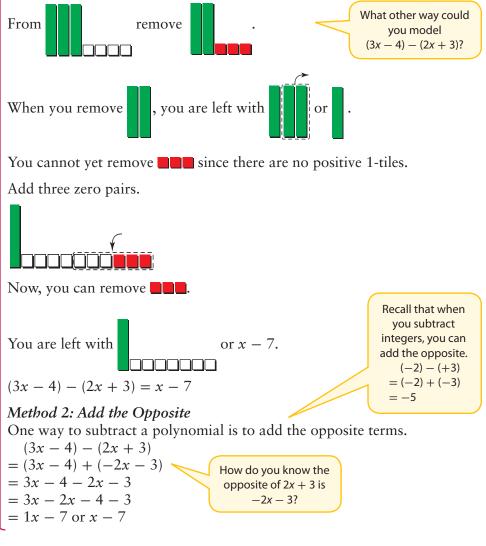
Subtract 2x + 3 from 3x - 4. Simplify your answer by combining like terms.

Solution

(3x - 4) - (2x + 3)



Method 1: Use a Model You can use algebra tiles to model each polynomial.





To learn more about adding and subtracting polynomials, go to www.mathlinks9.ca and follow the links.

Show You Know

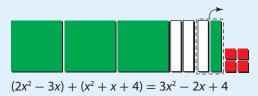
- a) Simplify the following expression. Model your solution. (2x - 3) - (-x + 2)
- **b)** Subtract and combine like terms. $(5x^2 - x + 4) - (2x^2 - 3x - 1)$

Key Ideas

• You can add or subtract polynomials. You can use models to help simplify the expression.

$$(2x^2 - 3x) + (x^2 + x + 4)$$

Group like terms. Remove any zero pairs.



• The opposite of a polynomial is found by taking the opposite of each of its terms.

-3x + 3x = 0x or 0, so it

does not need to be

included in the answer.

The opposite of $-3x^2 + x + 1$ is $3x^2 - x - 1$.

• To subtract a polynomial, you can add the opposite terms.

```
(6x<sup>2</sup> - 3x + 4) - (x<sup>2</sup> - 3x + 2)
= (6x<sup>2</sup> - 3x + 4) + (-x<sup>2</sup> + 3x - 2)
= 6x<sup>2</sup> - x<sup>2</sup> - 3x + 3x + 4 - 2
= 5x<sup>2</sup> + 0x + 2
= 5x<sup>2</sup> + 2
```

Check Your Understanding

Communicate the Ideas

- Jeanette and Tim find the answer to (3x² − 5x) − (4 − 2x). Jeanette claims the simplified answer has three terms. Tim says it has only two terms. Who is correct? How do you know?
- 2. What is the opposite of $-x^2 + 2x 3$? Use diagrams and then use symbols to determine the answer. Which method do you prefer? Why?
- 3. Identify any errors in Mei's work and correct them.

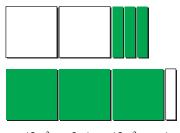
 $(-2x^{2} + 7) - (3x^{2} + x - 5)$ = $(-2x^{2} + 7) + (-3x^{2} - x + 5)$ = $-2x^{2} - 3x^{2} - x + 7 + 5$ = $5x^{2} - x + 12$

4. Create a situation in which the polynomials 3x + 2 and 5x - 1 are involved. In your situation, what does (3x + 2) + (5x - 1) represent?

Practise

For help with #5 to #7, refer to Example 1 on page 191.

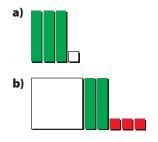
5. Which addition statement does the diagram model?



- **A** $(2x^2 3x) + (3x^2 x)$
- **B** $(-2x^2 + 3x) + (3x^2 + x)$
- **c** $(-2x^2 + 3x) + (3x^2 x)$
- **6.** Add the polynomials.
 - a) (3x 4) + (2x 3)
 - **b)** $(-a^2 3a + 2) + (-4a^2 + 2a)$
 - c) (5p + 5) + (5p 5)
 - d) $(2y^2 15) + (6y + 9)$
- **7.** Perform the indicated operation and simplify by combining like terms.
 - a) (-3x + 4) + (6x)
 - **b)** (3n 4) + (7 4n)
 - c) $(2b^2 3) + (-b^2 + 2)$
 - d) $(5a^2 3a + 2) + (-4a^2 + 2a 3)$

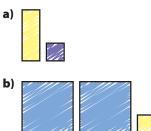
For help with #8 to #12, refer to Example 2 on pages 192–193.

8. What is the opposite of the expression represented by each diagram? Express your answer using both diagrams and symbols.



9. Let represent x^2 , represent x,

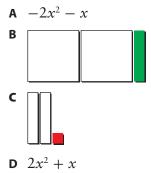
and zepresent 1. The same diagrams in yellow represent negative quantities Determine the opposite of the expression represented by each diagram. Use both diagrams and symbols to express your answer.



- **10.** What is the opposite of each expression?
 - a) -9xb) 5d + 6c) $-2x^2 + 3x - 5$
- **11.** What is the opposite of each expression?
 - **a)** 3*x* − 7

b)
$$4g^2 - 4g + 2.5$$

- c) $v^2 + 8v 1$
- **12.** Which of the following represents the opposite of $2x^2 x$?



For help with #13 to #15, refer to Example 3 on page 194.

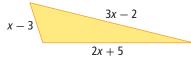
- **13.** Draw a diagram to model the subtraction statement $(-3x^2 + 4x) (-2x^2 x)$.
- **14.** Simplify by combining like terms.
 - a) (2x 3) (5x 1)b) $(-3b^2 - 5b) - (2b^2 + 4b)$ c) (5 - 6w) - (-2 - 3w)
 - **d)** $(m + 7) (m^2 + 7)$

15. Subtract.

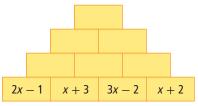
a) (8c - 3) - (-5c)b) $(-3r^2 - 5r - 2) - (r^2 - 2r + 4)$ c) $(y^2 - 5y) - (2y - y^2)$ d) $(6j^2 - 4j + 3) - (-2j^2 - 5)$

Apply

16. A triangle has the dimensions shown.



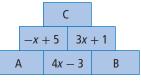
- a) What does (x 3) + (3x 2) + (2x + 5) represent?
- **b)** Simplify the expression in part a).
- c) If *x* has a value of 5, what is the perimeter of the triangle? Did you use the expression in part a) or part b) to find this answer? Why?
- 17. Complete the addition pyramid. Find the value in any box by adding the expressions in the two boxes immediately below it.



- 18. In Langley, British Columbia, you can rent a backhoe for \$399 per day and a bulldozer for \$550 per day. It costs \$160, round trip to move each piece of equipment back and forth to the job site.
 - a) Write an expression for the total cost of renting the backhoe, before tax. Include transportation to and from the job site. What does your variable represent?
 - b) What is an expression for the total cost of renting and moving the bulldozer? Use the same variable as in part a).
 - c) What is an expression for the cost of renting both a backhoe and a bulldozer? Give your answer in its simplest form.
 - **d)** What is an expression for the difference in cost between renting the backhoe and the bulldozer? Give your answer in simplest form.



19. Consider the addition pyramid shown below.



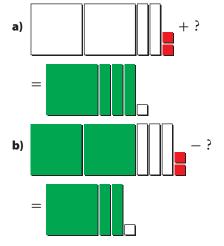
- a) Write an expression for box C. Do not simplify.
- **b)** Show how you can find the value for boxes A and B. Simplify your answers.

- **20.** The cost to print *n* copies of a book is 15n + 2000 dollars. The cost to ship *n* copies of the book is 2n + 150 dollars.
 - a) What is an expression for the total cost to print and ship *n* copies of the book?
 - **b)** What is the actual cost to print and ship 600 copies of the book?
 - c) What does (15n + 2000) (2n + 150) represent? Find a simpler expression for this subtraction statement.
- **21.** Describe any errors in Jorge's work and how you would correct each one.

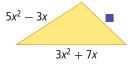
$$(4p2 - p + 3) - (p2 + 3p - 2)$$

= 4p² - p + 3 - p² - 3p - 2
= (4p² - p²) + (-p - 3p) + (3 - 2)
= 3p² - 3p + 1

- **22.** Simplify by combining like terms.
 - a) (6x 7) + (3x 1) + (x 4)
 - **b)** $(3a^2 4a) + (3a 5) (a^2 1)$
 - c) $(4t^2 t + 6) (t^2 + 2t 4) + (2t^2 3t 1)$
 - d) (2x 1.8) (3.4x 2.1) (0.9x 0.1)
- **23.** Replace each question mark with algebra tiles to make a true statement.



24. The perimeter of the triangle shown is $12x^2 + 6x$, in metres. Find a polynomial representing the missing side length.



25. Your student council plans to thank25 students and staff who have made special contributions to the



school. Wooden plaques cost \$17.95. It costs \$0.12 per letter to engrave a message on the plaque. All costs are before tax.

- a) Write an expression for the cost of engraving 25 plaques with the following message and the name of your school. *Thank you for your generous support.*
- **b)** Write an expression for the cost of buying and engraving the plaques.
- c) Write an expression for the cost of buying and engraving 25 plaques with an unknown number of letters.
- d) Show how to use the addition of polynomials to find the cost of 50 plaques if each plaque has the same number of letters and numbers.

Extend

- **26.** Kiesha's dad is a Haisla artist. He makes his own prints and sells them on the Internet. He will ship the prints to purchasers anywhere in Canada. For large prints, he charges \$30 to ship one print plus \$7 for each additional print. For small prints, he charges \$20 for one print plus \$5 for each additional print.
 - a) How much does her dad charge to ship two large prints?
 - **b)** How much does he charge to ship four small prints?
 - c) Write an expression to show how much he charges to ship an unknown number of large and small prints.

27. The length of the picture shown is 15 cm more than its width. The picture frame has a width of 4 cm. What is the minimum length of material needed to make the frame for this picture? Give your answer as a simplified expression.



- **28.** A small manufacturer makes air quality monitoring kits for home use. The revenue, in dollars, from the sale of *n* kits can be shown by $-n^2 + 3600n$. The cost, in dollars, to make *n* kits is represented by $-3n^2 + 8600$. The manufacturer makes a profit if the cost subtracted from the revenue is positive.
 - a) Write an expression to find the profit. Simplify your answer.
 - b) Estimate and then calculate if the manufacturer will make a profit or suffer a loss after selling 20 test kits. Explain.
- **29.** Simplify (2x + 4x + 6x + 8x + ... + 2006x + 2008x) (x + 3x + 5x + 7x + ... + 2005x + 2007x).
 - Math Link

Try this number trick several times.

- a) How can you find the original number from the number in the last step?
- **b)** Use algebra to show why this number trick works.
- c) Find or make your own number trick. Use algebra to show why it works.

30. Mary is sewing two wall hangings. The length of one wall hanging is 56 cm greater than its width. The length of the other wall hanging is 15 cm greater than its width. Each of them has the same width. She is going to add a trim strip around each wall hanging. What is the total minimum length of trim she will need for both wall hangings?



"Camp, With Animals Nearby" by Annie Taipanak (1931-) of Baker Lake and Rankin Inlet, Nunavut.



"Hunting Caribou by Kayak" by Tobi Kreelak (1977-) of Baker Lake, Nunavut.

Guess a Number

Step 1 Pick a number. **Step 2** Add 5. **Step 3** Double the sum. **Step 4** Subtract 10.

Chapter 5 Review

Key Words

For #1 to #6, choose the letter that best matches each description. You may use each letter more than once or not at all.

 1. 3w is a like term
 A -3x + 1

 2. has three terms
 B -4d + 3

 3. monomial
 $C \ 1 - 3x^2$

 4. opposite polynomial to 3x - 1 D -w

 5. polynomial of degree 2
 F -3x - 1

G 3f - 1

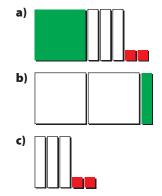
6. contains the constant term 3

5.1 The Language of Mathematics, pages 174–182

- **7.** For each expression, identify the number of terms and whether the expression is a monomial, binomial, trinomial, or polynomial.
 - a) $5 p + px p^2$
 - **b**) 3*f* − *q*
 - **c**) −2*a*
 - d) $5xy 27x^2 + 2$
- **8.** What is the degree of each polynomial? Explain how you found your answers.
 - **a)** $6x^2$
 - **b)** ab 7a + 3
 - **c)** 3 − y
- 9. Provide an example of each of the following.
 - a) binomial
 - **b**) polynomial with three terms
 - c) polynomial of degree 2
 - d) monomial that is a constant term
- **10.** Model each expression.

a) 1 - v b) $3x^2 - 2x + 1$

11. What expression is shown by each model?



12. You are selling used video games for \$10 and used books for \$4. The expression 10x + 4yprovides a general statement of the value of your sales before tax.



- a) What do the variables x and y represent?
- **b)** How much money do you receive if you sell 6 video games and 11 books?
- c) Write a new expression for selling DVDs at \$7.25 and CDs at \$5.

5.2 Equivalent Expressions, pages 182–189

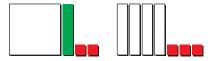
- 13. Tom claims that 4 and 4x are like terms since they both contain 4. Do you agree? Explain.
- **14.** For each expression, identify the coefficient, variables, and exponent for each variable.

a)
$$8xy^2$$
 b) $-c^2$ c) -4

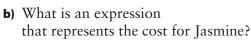
15. Identify the like terms in each group.

a)	7 <i>r</i>	3 <i>s</i>	$-s^{2}$	7	4 <i>rs</i>	-8	S
b)	$-2x^{2}$		3xy	x^2	5.3y	2	3xy

- **16.** Explain how you can tell like terms by looking at them. Give four different sets of examples with at least three like terms in each set.
- **17.** The following diagrams represent terms in an expression. Draw a new diagram with like terms together and write an expression for the simplified answer.



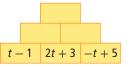
- **18.** Use materials or diagrams to model each expression and show how to combine like terms. What is the simplified expression?
 - a) 3 2x + 1 + 5xb) $2x^2 - x + 4 - x^2 + 5x - 1$
- **19.** Combine like terms.
 - **a)** 4a + 3 + 9a + 1
 - **b)** $2b^2 5b 4b^2 + 8b$
 - c) 1 c + 4 + 2c 3 + 6c
- **20.** Draw a shape with a perimeter represented by (4x) + (3x 1) + (x + 3) + (x 2), where each quantity in parentheses represents the length of a side. Find a simpler expression for the perimeter by combining like terms.
- 21. Kara and Jasmine go to Splash-o-mania water park. The entrance fee is \$20.00. Kara rents a locker for \$1.50 per hour. Jasmine rents a tube for \$3.00 per hour.
 - a) What is an expression that represents the cost for Kara?



c) What is a simplified expression for the total cost for both Kara and Jasmine to stay at the water park together for any number of hours?

5.3 Adding and Subtracting Polynomials, pages 190–199

- 22. a) Simplify each of the algebraic expressions. $(4x - 3) + (x - 1) \quad (4x - 3) - (x - 1)$
 - **b)** How are the processes similar? How are they different?
- **23.** Is $2x^2 3x$ the opposite of $3x 2x^2$? Show how you know.
- **24.** For each of the following expressions, what is the opposite?
 - **a)** -3 **b)** 7-a **c)** $x^2 2x + 4$
- **25.** a) Show how to simplify the following addition. Use two different methods. (3p + 4q 9) + (2 5q p)
 - **b)** Which method do you prefer? Why?
- **26.** Combine like terms.
 - a) (-p + 7) + (4p 5)
 - **b)** $(a^2 a 2) (5 3a^2 + 6a)$
- **27.** Complete the subtraction pyramid. Find the value in any box by subtracting the two expressions in the boxes immediately below it. Subtract in order from left to right.

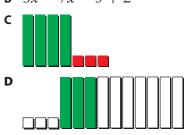


- **28.** An end-of-year class party has a fixed cost of \$140 to cover printing, decorations, and awards. In addition, it costs \$12 to feed each person who attends.
 - a) What is an expression for the total cost of the party? What does your variable represent?
 - **b)** Create a short scenario to generate an addition or subtraction question with polynomials. Simplify by combining like terms.

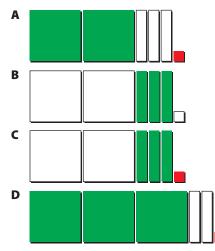
Chapter 5 Practice Test

For #1 to #6, select the best answer.

- **1.** Which polynomial is of degree 1?
 - **A** 3 7x **B** xy - 1 **C** 5x - 3xy**D** $x^2 - 5x + 2$
- 2. Which expression does not have zero as a constant term?
 - **A** -5x **B** k+8 **C** $y^2 - 2y$ **D** ab+b-c
- **3.** Which of the following is not equivalent to 3x 5 + 2 7x?
 - **A** -4x 3
 - **B** 3x 7x 5 + 2



4. Which set of diagrams represents $3x - 2x^2 + 1$?



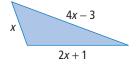
- 5. Which expression is a trinomial?
 - **A** abc^3
 - **B** 3mn
 - **c** $ef + g^2$
 - **D** -1 x + c
- 6. Which expression is the opposite of $-2k^2 + 3k 1$?
 - **A** $-1 3k + 2k^2$
 - **B** $1 3k + 2k^2$
 - **c** $1 3k 2k^2$
 - **D** $-1 3k 2k^2$

Complete the statements in #7 and #8.

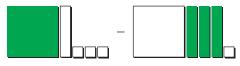
- 7. When you combine like terms, the expression $2t^2 5 8t^2 4$ becomes
- **8.** In the monomial $-q^2$, the value of the coefficient is **1**.

Short Answer

- **9.** Draw a diagram to represent $x^2 2x$.
- **10.** Create a single polynomial with
 - two terms
 - two variables
 - degree 2
 - a constant term
- **11.** What is an expression, in simplest form, for the perimeter of the triangle?



12. Write an expression to represent what the diagrams show. Then, simplify.



13. Simplify. Use models for at least one of the expressions. Show your work.

a)
$$(2x^2 - 8x + 1) + (9x^2 + 4x - 1)$$

b) (4 - 6w) - (3 - 8w)

Extended Response

- 14. The number of peanuts two squirrels bury can be represented by 4n + 7 and 5n - 1, respectively.
 - a) Write and simplify an expression for the number of peanuts both squirrels bury.
 - **b)** What could the expression (5n - 1) - (4n + 7) represent?
 - c) What is a simpler expression for (5n-1) - (4n+7)?

- **15.** The cost for a birthday party at Big Fun Bowling is \$100 for up to ten children, plus \$5 per pair of bowling shoes. To rent the party room, the cost is \$20, plus \$4 per child for pizza.
 - a) What is an expression for the cost of bowling for up to ten children?
 - **b)** What is an expression for the cost of pizza in the party room for up to ten children?
 - c) What is a simplified expression for the total cost of up to ten children going bowling and having pizza in the party room?
 - d) Estimate, then calculate, the cost of nine children going bowling and having pizza in the party room.



Math Link: Wrap It Up!

You are an illusionist who is about to amaze your audience with a number trick. However, before you try the trick, you need to know how it works.

- a) Try the trick, Guess a Number, several times. What do you notice about the middle digit of the number in step 4?
- **b)** What do you notice about the other two diaits?
- c) How does the information from parts a) and b) help you to understand this number trick?
- d) Make up a number-guessing trick. Show how algebra can help explain your number trick.

Guess a Number

Step 1 Tell someone to write down a three-digit number with no repeating digits. During the entire trick, do not look at what the person writes.

Step 2 Have the person arrange the digits in decreasing order.

Step 3 Ask the person to arrange the same three digits in increasing order.

Step 4 Tell the person to subtract the number in step 3 from the number in step 2.

Step 5 Ask the person to circle one number in the difference.

Step 6 Ask what the other two digits are. Identify the digit that was circled.

